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THE NUMBER OF γ-RAYS EMITTED PER SECOND FROM RADIUM B AND C IN EQUILIBRIUM WITH A GRAM OF RADIUM AND THE NUMBER EMITTED PER ATOM DISINTEGRATING

By Alois F. Kovarik

Abstract

Number of γ rays emitted by RaB and RaC per atom disintegrating.—γ-rays entering a metal produce β-ray emissions. By a β-ray emission is meant the simultaneous emission of one or more β-rays, initially in the general direction of the exciting γ-rays. Those β-ray emissions which passed through the metal window front of a counting chamber were counted, using an automatic registering device previously described. Different thicknesses of various metals were used as fronts. Corrections for scattered γ-rays and for β-rays from the walls of the chamber were determined experimentally. A standard 1.305 mg of radium was used, placed between the poles of an electromagnet. The coefficient of absorption of the γ-rays was determined from corrected counts with different thicknesses of absorbing metal placed between the source and the counting chamber but nearer the source, and the coefficient of absorption of the β-rays from a determination of the thickness of the metal cover which gave the maximum number of counts. From these coefficients and the geometrical constants, the total number of emissions which would have been produced if all the γ-rays from RaB and RaC had been absorbed in the metal was determined for Al, Cu, Sn, Pt, and Pb and found the same for all within a mean variation of ± 3 per cent, viz. 7.28×10^{10} per gm Ra per sec. equal, within experimental error, to twice 3.57×10^{10} which is the number of atoms of each disintegrating per second. Evidently this proves that each radioactive transformation results in the emission of a γ-ray entity which sooner or later produces a β-ray emission from a single atom. For the hard γ-rays, capable of penetrating 1.55 cm of lead, the number of counts, corrected for absorption and scattering, came out 76 per cent of the total. Ionisation experiments, repeated by the author, give about 73 per cent, the smaller figure being due to the smaller relative ionising power of the fast β-rays. Since γ-rays are entities, the counts seem to be more significant.

Coefficient of absorption of γ-rays from RaB and RaC and for secondary β-rays, determined from counts.—For all the γ-rays the values found for Al, Cu, Sn, Pt, and Pb, are 0.121, 0.38, 0.36, 1.40 and 0.78 per cm, and for the hard γ-rays 0.11, 0.32, 0.29, 0.81 and 0.47 respectively. For the β-rays in the metal in which they were produced (1) by all the γ-rays: 38.6, .160, 84.3, 374 and 200 per cm, (2) by hard γ-rays: 16.2, 58.1, 52.3, 215 and 104.

Introduction

THE γ-rays from radioactive substances and x-rays have been looked upon as entities from the earliest days when attempts were made to explain them either as electromagnetic phenomena or later as corpuscular phenomena. That γ-rays are continuous waves has no experimental basis. From all experimental work one is forced to conclude that γ-rays, if electromagnetic waves, are discrete trains of waves, and therefore definite entities. Whether the γ-rays are the result of the expulsion of a β-particle from the nucleus or whether the expulsion of β-particles is due to γ-rays as is suggested by the recent work of Ellis,[1] the fact remains that between β-ray magnetic spectra and γ-ray spectra certain quantum relations hold. That γ-rays may be detected as entities was pointed out by Kovarik and McKeehan[2] in their statistical work on β-particles in considering the corrections due to natural causes and γ-rays.

It has been of interest from the very beginning to know the number of γ-ray entities emitted from an atom when it disintegrates. Eve[3] concluded from his experiments on ionisation by γ-rays from radium C that the γ-radiation contains twice as much energy as the β-radiation. Moseley[4] measuring secondary β-radiation due to γ-rays concluded on the basis of Bragg's corpuscular theory that the nearest whole number expressing the γ-rays per atom disintegrating is two. Rutherford and Robinson's[5] determination of the heating effects of the various radiations from radium and its products in equilibrium is 4.7 calories per hour per gram of radium for the β-rays of radium B and C and 6.4 for the γ-rays, or 1.36 times as much energy in the γ-rays as in the β-rays of radium B and C. In the preliminary results of the writer[6] using the counting method the value obtained for γ-rays from radium B and C per second per gram of radium was 7×10^{10}. The work of Hess and Lawson[7] then came (belatedly owing to the war) to the notice of the writer and stopped further work temporarily. The results obtained by Hess and Lawson are for radium C 1.43×10^{10} and for radium B 1.49×10^{10} per second per gram of radium. These values are so much below those obtained by the author and below what one might expect if a whole number of entities is associated with one disintegrating atom that the work was recently resumed and a record of this work is reported in this paper. It was

[1] Ellis, C. D., Proc.Roy. Soc. A **101,** 1 (1922)
[2] Kovarik and McKeehan, Phys. Zeits. **15,** 1 (1914)
[3] Eve, Phil. Mag. **22,** 551 (1911)
[4] Moseley, Proc. Roy. Soc. A **87,** 250 (1912)
[5] Rutherford and Robinson, Phil. Mag. **25,** 312 (1913)
[6] Kovarik, Proc. Nat. Acad. Sci. **6,** 105 (1920)
[7] Hess and Lawson, Sitzb. Akad. Wien. **125,** 585 (1916)

thought important to determine experimentally all of the constants used in the calculations and this was done except for the coefficient of absorption in air. The value obtained is 7.28×10^{10} γ-rays per second from radium B and radium C in equilibrium with one gram of radium, or one γ-ray per atom disintegrating.

Theory

Let us suppose that a beam of γ-rays is incident on a plate of matter. The β-rays which are produced are produced at the expense of the absorbed γ-ray energy. Let us look at a γ-ray entity as a train of waves whose energy is a definite quantum. This energy may be all used up in the substance into which the γ-ray enters either to produce (a) one β-particle or (b) several simultaneously from one atom or (c) several at intervals of time from different atoms. In the first two cases the ionisation produced by the β-particle or β-particles may be looked upon as if produced by one β-ray at one instant, and statistically, as one event due to one γ-ray. In the third case the number of β-ray events produced by one γ-ray would be greater than one. From the experiments of C. T. R. Wilson[8] on x-rays and γ-rays, it would appear that both of the first cases are possible. There seems to be no experimental evidence for the third case; and from the results of these experiments it would appear that such a condition does not exist, i.e. that the energy of a γ-ray is really all used up to produce one β-particle or several β-particles simultaneously or so nearly simultaneously at one place that they may be regarded as such. It is on this hypothesis as a basis that the calculations are made. In the method used in these experiments the phenomenon of the production of a single β-particle or several β-particles simultaneously can be detected as a single event and, therefore, corresponds to the absorption of one γ-ray entity. If the initially produced β-particles are supposed to have the direction of the γ-rays,[9] the laws of absorption of the β-particles would be the same as for normal rays passing through the matter. If there is any deviation from this supposition it will not affect the calculations provided the coefficient of absorption of the β-rays produced by the γ-rays is determined.* This was done in these experiments.

[8] C. T. R. Wilson, Proc. Roy. Soc. A **104,** 1 and 192 (1923)

[9] Rutherford, Radioactive Substances etc., p. 276

*If any of the β-rays are scattered, as they are, or if the γ-rays produce β-rays, initially, in the opposite direction, as very likely they do, the coefficient of absorption will not be that of true absorption but of all the effects which cause a diminution of the number of emergent β-rays and, therefore, of the events counted.

Let $N \equiv$ the number of γ-ray entities from a given source S that are incident on the plate of matter per second;

$S \equiv$ the strength of the source;

$a \equiv$ the area of the plate of matter on which the γ-rays are incident;

$r \equiv$ the distance of the source from the plate of matter, this distance being great enough to consider the rays normal to the plate;

$d \equiv$ the thickness of the plate;

$\mu_1 \equiv$ the coefficient of absorption of the γ-rays in the material of the plate;

$\mu_2 \equiv$ the coefficient of absorption in the material of the plate of the β-particles produced by the γ-rays in the same material;

$n \equiv$ number of events detected beyond the plate caused by emergent single β-particles or several simultaneously produced β-particles due to N γ-rays.

Consider an element dx of the plate at a distance x from the incident side. Then the γ-rays entering dx (assuming the exponential law of absorption) $\equiv N' = N\epsilon^{-\mu_1 x}$ and $dN' = -N\mu_1\epsilon^{-\mu_1 x}\,dx$, so that the γ-rays absorbed in dx are $N\mu_1\epsilon^{-\mu_1 x}\,dx$ and these give rise to, say, dn' β-rays. The β-particles are absorbed exponentially and therefore the emergent β-particles $\equiv dn = \epsilon^{-\mu_2(d-x)}\,dn' = \epsilon^{-\mu_2(d-x)} N\ \mu_1\ \epsilon^{-\mu_1 x}\,dx$

$$\text{Hence} \quad n = N\mu_1\ \epsilon^{-\mu_2 d}\int_0^d \epsilon^{(\mu_2-\mu_1)x}\,dx\ = \frac{\mu_1}{\mu_2-\mu_1} N\ (\epsilon^{-\mu_1 d} - \epsilon^{-\mu_2 d}) \qquad (1)$$

and

$$N = n\ \frac{\mu_2-\mu_1}{\mu_1}\ \frac{1}{\epsilon^{-\mu_1 d} - \epsilon^{-\mu_2 d}}.$$

Since N represents the γ-rays per second from a given source S incident on a plate of area a which is situated at a distance r from the source, then if $\Gamma_1 \equiv$ total γ-rays per second from a unit source at a distance r, uncorrected for absorption by air and walls of vessel containing the source,

$$\Gamma_1 = n\frac{\mu_2-\mu_1}{\mu_1}\ \frac{1}{\epsilon^{-\mu_2 d} - \epsilon^{-\mu_1 d}}\ \frac{4\pi r^2}{aS} \qquad (2)$$

and if $\Gamma_0 \equiv$ total γ-rays emitted per second by a unit source,

then $\Gamma_0 = \Gamma_1 \times$ reciprocal exponentials, correcting for absorption by the air and walls of the vessel containing S. (3)

Eq. (1) on the right hand side shows a difference of two exponentials. With an increase in d there is an increase in the production of β-particles but also a rapid decrease of n due to the absorption of the β-particles. The relation between n and d shows a maximum for n for a certain value of d. If $D \equiv$ the thickness of the plate for maximum value of n, the condition for this maximum is found from Eq. (1) to be

$$\mu_1\ \epsilon^{-\mu_1 D} = \mu_2\ \epsilon^{-\mu_2 D} \qquad (4)$$

The Eq. (2) then becomes

$$\Gamma_1 = n\frac{1}{\epsilon^{-\mu_1 D}}\frac{4\pi r^2}{aS} = n\frac{\mu_2}{\mu_1 \epsilon^{-\mu_1 D}}\frac{4\pi r^2}{aS} \qquad (2')$$

In the experimental arrangement we may either use any thickness d of the plate and solve for Γ_1 by Eq. (2) or we may use the definite thickness D and solve by the simpler equation (2').

Apparatus

The apparatus used for the detection and counting of the β-particles has been described in previous papers[10] on α, β and γ-rays. The events to be counted were automatically registered by a device described in this journal.[11] The counting chamber in most of the experiments was a cylinder 5.5 cm long and 0.81 cm in radius. The axis of the chamber was in the direction of the source. The circular front of the chamber was a plate of the material in which the γ-rays were absorbed and β-particles produced. In the early experiments the entire chamber was of the same material and thickness as the front, but in the final experiments the chamber cylinder was of paper (d = .0384 cm), India inked on the inside, or of aluminium (d = .0365 cm), and the front was a circular plate made of different material in different experiments. In some experiments, particularly those in which the chamber used was of the same material and thickness as the front, the counting chamber was in the open air; in the final experiments, it was in a lead block cast to house the chamber. The protective walls were 3 to 4 cm thick and the cylindrical opening for the chamber was about 3.4 cm in diameter and was lined with thick paper. The metals used for the fronts were Al, Cu, Sn, Pt, Pb, each in several thicknesses.

The source of γ-rays was radium B and radium C in equilibrium with radium which was enclosed in a glass tube. Radium of strength from 0.15 mg to 5.002 mg was used but in the final experiments the laboratory standard of 1.305 mg was the source of γ-rays. When the hard γ-rays were studied the radium tube enclosed in a brass tube was placed in a lead cylinder of 1.55 cm thickness which was provided with lead caps at the two ends. The radium tube was set up vertically between the poles of a large Pye magnet to deflect the direct β-radiation coming from the source in case of the total radiation, and the β-radiation from the lead in case of the hard radiation.

[10] Geiger, Verh. d. D. Phys. Ges. 15, 534, 1913
Kovarik and McKeehan, Phys. ZS. 15, 434, 1914; Phys. Rev. **6**, 426, 1915 and **8**, 574, 1916.

[11] Kovarik, Phys. Rev. **13**, 272, 1919

Procedure and Precautions

The quantities to be determined experimentally are n, μ_1, μ_2, d and r. The thickness of the plate d and the distance r were easily measured. The coefficients of absorption and n depend on counting the β-ray events under the various necessary conditions.

The precautions and corrective measures to be taken to obtain correct results for the counts are of great consequence. The γ-rays passing from the source in all directions are scattered not only by the air but by all objects in the neighborhood. This scattering effect is very appreciable. If the counting chamber is left in the open, not only do direct rays enter through the front, but also scattered γ-rays, both through the front and through the cylindrical sides of the chamber. The latter effect of the scattered rays can be minimized by housing the chamber in a block of lead having walls of considerable thickness. The rest of the effect due to scattered radiation along with any natural effect such as natural discharges from the point or effects due to penetrating radiation from any source other than that used, was corrected for in the following way. Lead plates aggregating 19 cm in thickness were placed in the path of the direct rays at a position such that the scattered radiation effect was a maximum. There is a range of positions for which the scattered effect has about the same (maximum) value. The position nearest to the source was used for reasons given below. If the lead plates are next to the chamber then on account of their size the scattered radiation coming to the chamber from a considerable solid angle is cut out in this observation while this scattered radiation will enter when counts are taken for the direct plus the scattered radiation effect without the use of the lead plates. If the lead plates are next to the source, then radiation from the source in directions other than that toward the counting chamber is also absorbed and therefore cannot be scattered as is the case when counts are taken without the lead plates. In both of these instances, therefore, the correction for scattered radiation would be too small. In a position between the source and the counting chamber both of these difficulties are obviated unless the plates are very large or the distance of the source from the counting chamber (when a weak source is used) is small. For the final experiments the source was 310 cm from the chamber and the lead plates, about 12 cm$\times$12 cm, were placed at 90 cm from the source for this correction reading, For a distance of 310 cm between the source and the counting chamber the lead plates could be placed as much as 100 cm nearer the counting chamber than they were and a correction reading still be obtained of about the same maximum value. Now, in experiments on absorption of γ-rays, absorbing

plates of various thicknesses are used and if placed near the counting chamber, the β-particles produced in them and passing in the direction of the γ-rays would enter the counting chamber and produce records similar to those for the γ-rays and the effect counted would not be due alone to γ-rays which are not absorbed, but also partly to these β-particles. For this reason the same position was used for the lead plates and for the absorbing plates in absorption experiments, and this position was as far away from the counting chamber as possible. In this way reliable correction readings for the scattered radiation and for the count of the γ-rays passing through the absorbing plates in absorption experiments were obtained.

Since the counting chamber is a cylinder of some length and the discharge point cannot readily be placed in a position so that the intense electric field exists only between the point and the front but extends usually to the walls of the chamber as well, and moreover since some β-particles are produced in the cylindrical wall of the chamber itself, it is necessary to standardize the position of the point and to make a correction for those β-particles that do not come from the front. On varying the distance between the point and the front of the chamber and thereby varying the surface of the chamber to which the intense field from the point spreads, the counts obtained are different. When the point is near the front the count is small but as the point is moved farther back the count rapidly increases until the surface includes the whole of the front, and then increases less rapidly as the surface includes more and more of the cylindrical surface of the chamber. It is essential, therefore, to have the point always in the same position with respect to the front and to make corrections for effects due to β-particles which do not come from the front. If the point is well to the rear of the chamber, one would expect that the strong field lies mainly between the point and the sidewalls of the chamber, and that by separating the front from the chamber electrically one would obtain the same counts as when the front was a part of the chamber. This was tested experimentally. The point was set 4 cm from the front of a copper counting chamber and records made for a given radiation. The front was then set a millimeter in front of the chamber, and was earthed, the chamber being at about 2000 volts. The reading in the second case was only 0.3 per cent smaller than in the first case. It was, therefore, decided that a record taken with the front removed corrects for all β-particles produced elsewhere in the chamber than in the front. A further test on the effect of the modified field, with the front removed, was carried out as follows. Wire gauze fronts were made with 5, 10, and 15 fine copper wires uniformly arranged and

crossing at right angles and records were taken with each of them and the curve obtained was extrapolated to a condition with no wire front. The effect, of course, increased with an increase in the number of wires, due (a) to any increase of field to the front and (b) to β-particles produced by the γ-rays in the wires. The increase, as shown by the first test, is undoubtedly due mainly to the second cause. The extrapolated value corresponded to the count made with no front. In all the experiments from which data were collected, records were also made for "no fronts" for the hard rays alone and for all the rays from the source. This correction reading involved observations made throughout the series of experiments and in aggregate took several hours.

In taking records of counts, therefore, records were taken for (1) the primary and scattered radiation together, for (2) the scattered radiation alone (including natural effects of the point, etc.) and for (3) "no front" on the chamber. The second was always taken along with the first and ultimately the difference of the two readings was corrected for the mean value of the third. The net result represents the effect due to γ-rays acting on the front alone.

Two distinct sets of observations were made involving separate determinations of all the quantities involved, namely, for all γ-rays emerging from the glass tube containing the radium, which are designated as total or *heterogeneous γ-rays*, and for the γ-rays passing through the glass tube, a protecting brass tube and a lead cylinder 1.55 cm thick, which are designated as *hard γ-rays*.

The inverse square law assumed in the calculations was tested by obtaining records of counts when the radium was placed at various distances, 200 cm to 450 cm from the counting chamber. The net count of the β-ray events produced by the γ-rays incident on the front multiplied by the square of the distance gave substantially the same value for these distances. At 450 cm two radium specimens, 1.305 mg and 5.002 mg, gave net counts proportional to the strengths of the radium specimens. It may be assumed, from these tests, that the application of the inverse square law and the proportionality of net counts to the strength of the material is justifiable.

The effect of the area of the front on the net count of the phenomena registered was tested in two ways. In the first method three chambers of areas in the ratio of 1:1.91:4.1 were used and the registered counts corrected for scattered radiation and for "no front" effects were in the ratio of the areas. In the second method one chamber enclosed in a lead house was used. Two blocks of lead were placed so as to form a slit. The net counts obtained for different widths of slits were plotted

against the exposed areas of the front as the two blocks of lead were separated. The resulting curve was a straight line.

It was therefore concluded that the corrections as applied must give sensibly the correct readings for any position of the source and for the size of the chamber. Consequently, in the final experiments, only one distance r corresponding to a convenient position of the source relative to the chamber was used and the chamber was also selected of a convenient size having a fixed area of the front.

Coefficients of Absorption

The coefficient of absorption of the γ-rays was obtained from counts, corrected for scattering by air and objects and for natural effects, for different thicknesses of absorbing material, by plotting the logarithm of the number against the thickness. The values obtained are given in the table below under μ_1 and μ_1', for "hard" and "heterogeneous" γ-rays. The coefficients on the whole indicate harder rays than do values found by ionisation methods, which can readily be explained by the fact that in ionisation measurements the values depend not only on the number of γ-rays but also on the ionisation per cm of the β-particles produced by them and this increases with the decrease of velocity of the β-particles.

The coefficient of absorption of the β-particles produced by the γ-rays might be obtained in a manner similar to that followed by Bragg and Madsen[12] or by determining the thickness of the material giving maximum effect, i.e. the quantity D, and with the knowledge of D and μ_1 solving Eq. (4). It must be admitted that the experimental determination of D introduces the greatest experimental error into the calculations, but the calculation of μ_2 involves no approximations. It was this second method that was used. Determinations of D from three types of experiments were made, (1) using chambers of different thicknesses in open air and plotting the count against thickness; (2) housing a standard counting chamber in the lead block and using fronts of the chamber of different thicknesses; and (3) repeating Bragg and Madsen's experiments, with some modification in the measuring instruments, by the measurement of the ionisation current. These last experiments were repeated to find if the maximum appeared materially different from that obtained by the counting method inasmuch as the μ_2 seemed to come higher in value than that reported by Bragg and Madsen. The values of D so obtained were not materially different from those obtained by the counting method. The average of all values by all methods for

[12] Bragg and Madsen, Phil. Mag. **16**, 918 (1908)

each metal was used. The value of μ_2 was obtained from Eq. (4) by trial after changing the equation in the following way: $\mu_1\epsilon^{-\mu_1 D} = \mu_2\epsilon^{-\mu_2 D} = A$ and $\log_{10}\mu_2 - \log_{10}A - \mu_2 D\log_{10}\epsilon = \phi$ and giving values to μ_2 until $\phi = 0$, A being determined from μ_1 and D. Table I gives the values

TABLE I

The coefficient of absorption of γ-rays, μ_1, and the coefficient of absorption in a metal of the β-particles produced in the metal by γ-rays, μ_2

Metal	μ_1	D	μ_2	μ_1/ρ	$M\mu_1/\rho$	μ_2/ρ	$M\mu_2/\rho$
			Hard γ-rays				
Pb	0.47 cm	0.052 cm	104.3 cm	0.041	8.56	9.17	1900
Pt	0.81	0.026	215.5	.038	7.36	10.02	1960
Sn	0.29	0.10	52.3	.040	4.72	7.20	855
Cu	0.32	0.09	58.1	.036	2.28	6.51	414
Al	0.11	0.31	16.2	.041	1.10	6.00	163
			Heterogeneous γ-rays				
Pb	0.78	0.029	200	.069	14.2	17.6	3640
Pt	1.40	0.015	374	.065	12.7	17.4	3400
Sn	0.36	0.065	84.3	.049	5.86	11.6	1373
Cu	0.38	0.038	160	.043	2.71	17.9	1140
Al	0.121	0.15	38.6	.046	1.21	14.3	388

of μ_1, D, and μ_2. The values of μ_2 are higher than the values for corresponding primary β-particles from the same source, which would indicate a softer β-radiation than the primary. If all the energy of the γ-ray goes to the ejection of a β-particle, the β-particle will not have the total energy because some of the energy must be used to eject the electron from some inner ring of the atom. Although the primary β-rays and the γ-rays are undoubtedly connected by some relation, the secondary β-radiation produced in metals, coming as a later transformation, should be softer than the primary radiation.

The columns μ/ρ and $M\mu/\rho$ give respectively the mass coefficient of absorption and the coefficient of absorption per gram atom for the γ-rays and for the β-rays produced by the γ-rays in the metal, where ρ is the density of the metal and M its atomic weight. If the values of $M\mu/\rho$ are plotted against the atomic number (Fig. 1), it will be found that the relation is not a linear one and that the relative increase in $M\mu/\rho$ with the atomic number is greater for the heterogeneous radiation, whether it be the γ-rays or the β-rays.

FINAL DISPOSITION OF THE APPARATUS AND OBSERVATIONS

Although several specimens of radium were at the disposal of the writer it was deemed advisable to use the laboratory standard whose radium content was separately determined with consistent results by Professor B. B. Boltwood and by the U. S. Bureau of Standards. It was considered best to place this specimen at such a distance so that

the counts obtained would be most convenient in number both for the hard γ-rays and for the total or heterogeneous γ-rays.

The standard 1.305 mg in its glass tube was placed between the poles of a strong magnet to deflect the primary β-rays and at a distance of 310 cm from the front of the counting chamber. The counting chamber n some experiments, made of paper India-inked on the inside, and in other experiments, made of thin aluminium, was enclosed in a lead block described above. The fronts used were of various metals and thicknesses given. The plates of lead (19 cm) to obstruct direct γ-rays were set at 90 cm from the source when counts were taken to correct for scattering and natural effects. Since this last count besides scattering involved also the natural effect due to the points used and these differed in sharpness, symmetry, etc. and therefore in their discharges, it was necessary to take a reading for this effect after each reading for the direct and

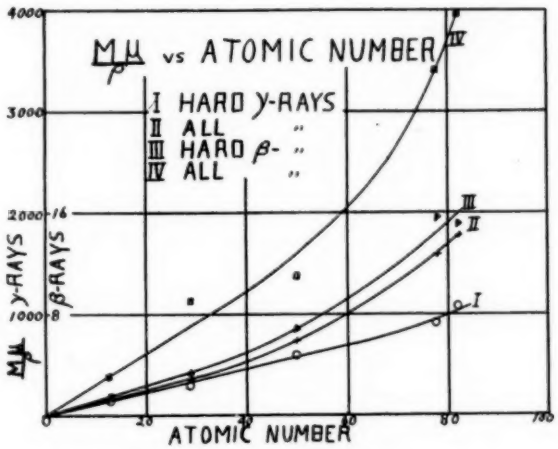

Fig. 1. Coefficients of absorption per gram atom for γ-rays and for the secondary β-rays.

scattered rays, etc; and the differences were finally averaged. The readings for any front used covered in total several hours of actual counts. Counts with the front removed were taken frequently during the whole time of the experiments. The correction so obtained was corrected by 0.3 per cent for the effect noted above when the front plate was earthed and set a millimeter in front of the chamber. For the heterogeneous rays this correction for "no front" amounted to 39.0 counts per minute and for the hard rays to 21.0 per minute. In the experiments with the hard γ-rays the radium tube was placed in a brass (protective) cylinder and a lead cylinder of 1.55 cm thickness and the whole was also

placed between the poles of the magnet deflecting the β-particles produced in the lead cylinder.

The final results are given in Table II. The third column gives the average of the difference in counts per minute when the direct and scattered γ-rays and natural effects were recorded and when the lead plates were set up to stop the direct γ-rays. The fourth column gives the net count per minute for the γ-ray effect produced in the metal of the front of the chamber, i.e. the value of the third column less the

TABLE II

Front of chamber	Thickness in cm	Total count less scattering per min.	Net count per min.	n Net per second	K per gm	$\epsilon^{-\mu_1 d} - \epsilon^{-\mu_2 d}$	Γ_1 per gm per sec.	Aver. Γ_1
				Total γ-radiation				
Pb	0.0324	75.2	36.2	0.603	11.47×10^{10}	0.973	7.10×10^{10}	
	.065	73.2	34.2	.570		.950	6.88	
								6.99×10^{10}
Pt	0.0040	64.5	25.5	0.425	11.95	0.747	6.80	
	.020	70.7	31.7	.528		.971	6.50	
								6.65
Sn	0.0238	75.0	36.0	0.600	10.47	0.857	7.33	
	.051	75.4	36.4	.607		.968	6.57	
								6.95
Cu	0.0248	61.7	22.7	0.378	18.85	0.972	7.33	
	.052	61.4	22.4	.373		.980	7.17	
								7.25
Al	0.0365	60.2	21.2	0.353	14.28	0.751	6.71	
	.1095	67.5	28.5	.475		.972	6.98	
								6.85
								Mean 6.94×10^{10}
				Hard γ-radiation				
Pb	0.0324	35.0	14.0	0.233	9.93	0.951	2.43	
	.065	36.6	15.6	.260		.969	2.67	
	.084	35.9	14.9	.248		.961	2.56	
								2.55
Pt	0.0040	28.8	7.8	0.130	11.90	0.575	2.69	
	.020	33.9	12.9	.215		.971	2.64	
								2.67
Sn	0.0238	35.7	14.7	0.245	8.05	0.705	2.73	
	.051	38.5	17.5	.292		.915	2.57	
								2.65
Cu	0.052	37.0	16.0	0.267	8.11	0.937	2.31	
								2.31
Al	0.0365	31.4	10.4	0.173	6.57	0.443	2.56	
	.1095	40.1	19.1	0.318		.818	2.56	
	.1042*	42.0	21.0	0.350	5.88	.806	2.56	
								2.56
								Mean 2.55×10^{10}

For total radiation: $\Gamma = \Gamma_1 \times \frac{1}{0.972} \times \frac{1}{0.981}$

$= 7.28\times10^{10}$ per gram per second

For hard rays: $\Gamma' = \Gamma_1' \times \frac{1}{0.482} \times \frac{1}{0.981} \times \frac{1}{0.988} \times \frac{1}{0.981}$

$= 5.56\times10^{10}$ per gram per second

* Thick Al chamber and $a = 2.30$ cm^2

"no front" correction; and the fifth column gives this count per second. Since it was thought best to have readings for at least two thicknesses of each material, Eq. (2) was used for calculating the number of γ-rays and was written in the form

$$\Gamma_1 = nK \frac{1}{\epsilon^{-\mu_1 d} - \epsilon^{-\mu_2 d}}$$

where $$K = \frac{\mu_2 - \mu_1}{\mu_1} \frac{4\pi r^2}{aS} = \frac{\mu_2 - \mu_1}{\mu_1} 4.49 \times 10^8/\text{gram},$$

since $a = \pi$ (0.81 cm)^2 = 2.06 cm^2, r = 310 cm and S = 1.305 mg. This value of K is given in the sixth column. In the last Al record, the value of K was obtained by taking a = 2.30 cm^2 since a different chamber was used made of thicker aluminium and with a larger window. The seventh column gives the difference $(\epsilon^{-\mu_1 d} - \epsilon^{-\mu_2 d})$. The value of Γ_1 is entered in the eighth column. The values of Γ_1 for different values of d for the same metal were averaged and entered in the last column. Since separate constants had to be obtained for the different metals the values of the Γ_1, using different metals for the front, have a greater significance than the values obtained for different thicknesses of the same metal. The values for each metal were therefore weighted alike. The average of all these values is given at the bottom of the table. For the total radiation this value of Γ_1 must be corrected for glass and for the intervening air; for the hard radiation, it must be extrapolated to zero thickness of lead on account of the lead cylinder and corrected for the brass tube, glass and intervening air. Since the coefficient of absorption of the hard rays in lead is 0.47 cm^{-1} Pb and the thickness of the lead cylinder was 1.55 cm, the exponential used in the extrapolation has the value 0.482. The coefficients of absorption in glass were found to be 0.117 cm^{-1} for the hard γ-rays and 0.28 cm^{-1} for the total γ-rays for small thicknesses of glass. The thickness of the glass wall was 1 mm, making the extrapolation exponentials 0.988 and 0.972 for the hard and total γ radiations, respectively. The brass (protective) tube was used only in connection with the hard γ-rays and the correcting exponential was found experimentally to be 0.981. The correction for absorption in air was taken the same for hard and total radiation because complete data are lacking on the coefficient of absorption in air and the value used is that of Chadwick,[13] namely, 6.0×10^{-5} cm^{-1} air at ordinary room temperature and pressure. The exponential used for the correction for 310 cm air has the value 0.981.

Using these corrections the final value obtained for the total γ-radiation emitted from radium B and C is $6.94 \times 10^{10} \times 1/0.9537 = 7.28 \times 10^{10}$

[13] Chadwick, Phys. Soc. London, **24**, 145 (1912)

per gram radium per second, and for the hard radiation $2.55\times10^{10}\times 1/0.4583=5.56\times10^{10}$ per gram radium per second. The γ-radiation from radium D and E is absorbed by the glass and air. Consequently the number 7.28×10^{10} represents the number of γ rays emitted per second by radium B and radium C in equilibrium with one gram of radium. Since at the equilibrium condition the rate of disintegration of atoms is the same for all substances in radioactive equilibrium, we may take one-half of the above number, namely, 3.64×10^{10} to represent the number of γ rays emitted by radium B and the other half by radium C in equilibrium with one gram of radium. Furthermore, since the number of radium C atoms disintegrating per second per gram of radium as found by α particle counts,[14] is 3.57×10^{10}, it follows that the number of γ-ray entities emitted by one atom of radium B or radium C is one, within 2 per cent, a difference which may be due to experimental error arising most probably for the most part in the determination of D.

Discussion of the Results

It was mentioned at the beginning that the γ-ray energy may possibly be absorbed partially at different places producing β-particles perhaps far apart from each other. In such a case the counts of all the β-rays would give a value greater than the number of γ-ray entities. The average number of counts at any one place would be greater than the number of γ-rays. The fact that the number of γ-rays per atom disintegrating deduced from the data of these experiments comes out one γ-ray per atom, within the possible experimental error, seems to negative this assumption because one would expect an integral number of γ-rays per atom. The writer concludes that a γ-ray gives up its energy to produce one β-particle or two or more β-particles from the same atom or molecule.

The number of hard γ-rays from radium B and C was found to be 5.56×10^{10} per gram radium per second. This is 76.4 per cent of the total radiation, thus leaving for the soft radiation 23.6 per cent of the total. The experiments of Tuomikoski[15] on the absorption coefficients of γ-rays by the ionisation method gave for the hard radiation 72.2 per cent and for the soft radiation 27.8 per cent. These experiments were repeated by the writer using more thicknesses of lead (below 1 cm thickness) than were used by Tuomikoski and using a strong magnetic field on the source. A large cylindrical block of lead, 25 cm in diameter and 7 cm thick, with a hole in the center in line with electroscope and the source, was used to protect the electroscope from scattered radiation.

[14] Rutherford, Phil. Mag. **28,** 320 (1914)
[15] Tuomikoski, Phys. Zeits. **10,** 372 (1909)

The electroscope 6 cm×6 cm itself had 2 cm of lead on all sides except the one facing the source. The absorbing lead was placed between the electroscope and the large cylinder, the space between being 6 cm, and for zero reading 20 cm of lead were placed between the lead cylinder and the source so that about 26 cm of lead were in the direct path of the γ-rays. The hard radiation came out to be 73.7 per cent and the soft radiation 26.3 per cent of the total, which agrees fairly well with Tuomikoski. The coefficient of the hard radiation (lead 1 cm to 4.2 cm) came out 0.51 cm^{-1} Pb against Tuomikoski's 0.50 cm^{-1} Pb, and for the heterogeneous radiation (0 to 1 cm Pb) came out 0.88 cm^{-1} Pb against 0.95 cm^{-1} Pb. Now, the coefficient of absorption by the counting method, as indicated in this work, is for the hard γ-rays 0.47 cm^{-1} Pb, and for the heterogeneous rays (0 to 1 cm Pb) 0.78 cm^{-1} Pb. It is evident that the ionisation experiments give a higher value for a coefficient of absorption than the counting experiments, a fact already alluded to, and indicate a softer radiation. This indication from ionisation experiments may be misleading because the ionisation is produced by the β-particles emitted from the metal by the γ-rays and the softer the β-rays are the greater is the ionisation produced by them. This fact readily explains the greater rise of the absorption curve at the beginning and, therefore, the steeper log I vs. d straight line, and hence the greater value obtained for μ. It also explains the somewhat greater percentage of the soft radiation obtained by the ionisation method. It must be remembered that in the counting method the effect counted is one event for the γ-ray whether it be of long or short wave-length, whereas the ionisation produced in a given space by the fast moving β-particle emitted by a high frequency γ-ray is less than that produced by a slower moving β-particle emitted by a γ-ray of lower frequency. The difference between the percentages of the hard γ-rays and soft γ-rays as obtained by the counting method, and the corresponding values obtained by the ionisation method is in the right direction and seems to be of the right value. Since the counting method gives the numbers of γ-rays, the percentages represent the ratios of the actual γ-rays of the two types of radiation respectively to the total radiation without the qualifications which must be applied to similar values obtained by measurements of ionisations. The figures in these ratios include the penetrating radiations and the soft radiations from both radium B and radium C.

Conclusion

It is concluded from this statistical study of the γ-rays from radium B and radium C that the γ-ray energy is absorbed at one place to produce

one or more β-particles at that place and that the number of γ-ray entities from radium B and radium C in equilibrium with one gram of radium is 7.28×10^{10} per second, or that one γ-ray entity is emitted when one atom of radium B or of radium C disintegrates.

The writer wishes to thank the Western Electric Company for the extended loan of the sensitive relay and audion used in the self-registering apparatus.

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SOFT X-RAYS FROM HEAVY ELEMENTS, TANTALUM TO GOLD

By Joseph C. Boyce

Abstract

Soft x-radiations from tantalum, tungsten, osmium, iridium, platinum and gold, excited by electron bombardment at voltages between 30 and 1500 volts, were studied by the use of Kurth's apparatus in which the photo-electric effect of the radiation per unit electron current is measured. In this range for each metal from 6 to 9 critical voltages were located to within about 5 per cent. The results are in fair agreement with the values Bohr and Coster predicted for the ionization potentials of the N-level electron orbits, by means of the combination principle, from the x-ray wave-lengths measured by Siegbahn and others. The limitations of the method are considered and also the extent to which agreement is to be expected between the two different methods of measuring the energies of these levels.

FOR all the elements for which sufficient high precision x-ray measurements had then been made, Bohr and Coster[1] predicted the energy levels which fall between those susceptible of direct measurements, either by x-ray or by optical means. The predictions were based on the principle of combination, the validity of which has been quite firmly established within the region ordinarily covered by x-ray spectroscopy. This investigation was undertaken in the attempt to measure directly some of the energies thus predicted.

The tube used was built by Kurth and described by him in his article[2] on the soft x-rays from certain lighter elements. It consists of a tungsten filament as source of electrons, a target of the metal to be studied, and a platinum photo-electric cell for the measurement of the radiation produced. (It also contained an arrangement to guard against the effect of ionization of any residual gas, but a lead to this, inside the tube, was accidently burnt out early in this investigation. However, it was found that the vacuum obtained was sufficiently good to continue without it.) The evacuating apparatus and electrical connections were reconstructed as they had been used by Kurth and the same kind of electrical measuring instruments were used. The procedure was the same except that it was found that the vacuum could be maintained by the charcoal in the liquid air traps without running the pump, once the tube had been evacuated and it and the charcoal thoroughly baked out. When its melting point

[1] Bohr and Coster, Zeits. f. Phys. **12**, 342 (1923)
[2] Kurth, Phys. Rev. **18**, 461 (1921)

permitted, occluded gases were removed from the target by heating it by electron bombardment.

Up to 120 volts a storage battery was used as the source of the target potential. For the higher voltages, in the measurements of tungsten and platinum, a motor generator, run from a large capacity storage battery, was used. It gave 1800 volts with great steadiness. The potential applied to the tube was regulated by a potentiometer arrangement of rheostats. Unfortunately the generator broke down after the two metals had been studied and the remainder of the work was carried out using a 375 volt motor generator and enough dry cells to give a range up to 1500 volts. The targets, with the exception of platinum and gold, were supplied by the General Electric Company through the courtesy of Dr. W. D. Coolidge. The osmium he had had compressed into a tablet. An attempt was made to use lead as a target, but in the absence of any cooling devices it softened under the radiant heat from the filament even before electron bombardment had been started.

The curves showing critical potentials were obtained as by Kurth by plotting against the exciting voltages the quotient of the electrometer readings divided by the thermionic currents. Changes of slope were taken to indicate critical potentials. Many of the changes were downward instead of upwards, but excepting one doubtful case the changes at the same voltage were always of the same sort during different runs on the same metal. Kurth has already pointed out that either kind of change of slope might mark the setting in of a new type of radiation. There is a possibility that a bombarding electron may successively ionize two or more atoms in the target. This would give critical potentials at integral multiples of other critical potentials. A few such cases are suspected and accordingly marked as second order. In the first table all the results are given, including lines marked doubtful and the suspected second orders.

TABLE I

Voltages at which new types of radiation appear.

Ta (73)	W (74)	Os (76)	Ir (77)	Pt (78)	Au (79)
38	32.8	37	36	44	62 ?
47	37.2	55	47	82	90.8
77	74	81	60	106 ?	257
180	360	212	91	206 ? s	445
377	403	343	112	548	548
514	583	600	188	737	890 s
666 ?	815 s	1422 ?	240	1509 s	1335
1370 ? s			376		
			582		
			1336 ?		

? means doubtful; s means suspected higher order.

In the study of the curves isolated critical potentials were easily observed and located to within five percent. This accuracy depends on the steadiness of the tube operation and the number of observations it is possible to make within a small range. But when several such bends in the line lie close together they seem to blend and the exact number of components and their individual positions are hard to determine. It is hoped to improve this resolution in further investigations, but in the present results certain changes of slope though clearly present have been very hard to locate with definiteness. This particularly applies to those that have been assigned to the N I, N II, and N III levels. The results have been arranged in the first table in volts and in Table II in the corresponding values of T/R, where T is the wave number (from the quantum relation) for the spectral term and R is the Rydberg constant. Corresponding predictions by Bohr and Coster and others by Hjalmar[3] are also shown as well as the names assigned by the former to these levels. The predictions for osmium and iridium are not found in the Bohr and Coster tables. I have calculated them for these elements, as well as for the others, by interpolating from them the value of L I and using with it the wave-length measurements given by Coster[4] and by Hjalmar[3] to find the N and O energy levels, but the wave numbers thus calculated show divergence from the more accurate values that far exceeds the divergence of the experimental and predicted values in Table II, so it has not seemed worth while to include them in the Table. In naming all the levels I have followed the new symbols of Bohr and Coster.[5]

The tabular comparison, I think, shows fair agreement with the predicted values, especially considering the uncertainty in the true values indicated by the lack of agreement between the results of the two workers. The agreement is especially good in the cases of platinum and tungsten, the two cases where the experimental conditions were most satisfactory, as noted above. Still we must not expect too great agreement between the x-ray predictions and these direct voltage measurements. In x-ray emission measurements the N levels are those of departure when the interior portion of the atom is in an ionized state, while in such experiments as this the interior portion of the atom is in a normal state. It has been pointed out by Campbell[6] and others that this should make certain differences in the relations of x-ray absorbtion limits and emission lines, which however are scarcely large enough

[3] Hjalmar, Zeits. f. Phys. **15,** 65 (1923)
[4] Coster, Zeits. f. Phys. **6,** 185 (1921)
[5] Bohr and Coster, loc. cit.[1] p. 347.
[6] N. R. Campbell, Structure of the Atom, p. 75, Cambridge University Press, 1923

TABLE II

Comparison of values of T/R computed from the critical potentials in Table I with those computed from x-ray data by the combination principle.

	T/R (Obs.)	T/R (B & C)	T/R (H)	T/R (C)	Level
Tantalum	2.80	2.5	2.5	1.0—1.1	N VI, VII
	3.46	2.6	2.6	—	O II, III
	5.67	5.3	—	—	O I
	13.7	17.2	16.1	15.6	N V
	—	18.0	—	15.9	N IV
	27.7	30.0	—	30.0	N III
	38.0	33.0	—	—	N II
	49.1	41.7	—	—	N I
	101.0 ? s	—	—	—	M
Tungsten	2.42	2.5	2.5	3.0—3.2	N VI, VII
	2.74	2.9	2.7	—	O II, III
	5.45	5.2	—	—	O I
	—	17.8	17.6	18.5	N V
	—	18.7	—	18.4	N IV
	26.5	—	—	—	—
	29.7	30.6	—	30.9	N III
	—	35.6	—	35.8	N II
	42.9	43.6	—	43.4	N I
Osmium	2.72			—	O III, IV
	4.05			3.6	N VII
	5.96			3.4	N VI
	15.6			20.1	N V
	25.2			21.3	N IV
	44.2			44.4	N I
	104.5 ?			—	M
Iridium	2.65			—	O III, IV
	3.46			4.0	N VII
	4.42			4.2	N VI
	6.70			—	—
	8.23			—	—
	13.8			21.3	N V
	17.7			22.4	N IV
	27.7			38.4	N III
	—			—	N II
	43.0			47.9	N I
	98.4 ?			—	M
Platinum				T/R (D & P)	
	—	0.4	—	0.4	O IV, V
	3.24	5.2	5.3	5.6 & 5.7	N VI, VII
	6.04	7.1	5.7	7.1	O I
	7.8 ?	8.6	—	3.8	O II, III
	15.2 ? s	23.2	—	23.2	N V
	—	24.5	26.2	24.1	N IV
	40.4	42.3	36.3	43.9*	N III
	—	48.7	—		N II
	54.2	52.5	—	53.2	N I
	111 s	—	—	—	M
Gold	4.56 ?	0.8	—	0.8	O IV, V
	6.68	6.4	6.7 & 7.1	5.4 & 6.7	N VI, VII
	—	7.8	—	7.7	O I
	—	8.3	—	6.1	O II, III
	18.9	25.0	25.2	25.0	N V
	32.8	26.4	—	25.8	N IV
	40.4	42.8	37.5	40.6	N III
	—	49.1	—	46.8	N II
	65.6 s	58.0	57.3	56.8	N I
	98.4 ?	—	—	—	M

B & C—Bohr and Coster; H—Hjalmar; C—Cork;
D & P—Bohr and Coster's results corrected by Duane and Patterson.
*—not resolved.

to be detected in the regions of shorter wave-lengths. The optical differences are well known and in this intermediate region, if the resolution were improved, the difference might easily be measured. Fowler[7] has shown that the optical spectra of trebly ionized silicon corresponds to an ionization potential of 44.95 volts and has since pointed out[8] that similar effects are bound to complicate the spectroscopy of the soft x-ray and far ultraviolet by adding to the single ionization of the intermediate levels the results of multiple ionization of the optical levels.

It is believed that the results reported in the present paper justify the hope that the method of soft x-rays may be further refined so as to give energy levels of electrons in the N and O levels with sufficient precision as to be of considerable value in spectroscopy and the theory of atomic structure.

In conclusion, I wish to express my thanks to Professor K. T. Compton whose initial guidance made this work possible, to Professor O. W. Richardson for his subsequent consideration and criticism of the results, and to Mr. C. C. Van Voorhis for his frequent assistance in repairs to the glassware. The work was done in the Palmer Physical Laboratory at Princeton, New Jersey, during the summer of 1923.

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October 26, 1923.

[7] Fowler, Proc. Roy. Soc. A **103,** 413 (1923)

[8] Fowler, discussion at Liverpool meeting of British Association September 1923; also see account published in Nature, Feb. 9, 1924, p. 219.

CORPUSCULAR THEORY OF THE DISTRIBUTION OF THE RECOIL ELECTRONS PRODUCED BY POLARIZED X-RAYS

By G. E. M. Jauncey

Abstract

In a previous paper a formula for the number of x-ray corpuscles scattered in a direction ϕ and crossing 1 cm^2 of a sphere of large radius R has been obtained for the case of the scattering of polarized x-rays. For every x-ray corpuscle scattered in this direction, an electron recoils in the direction ψ, where $(1+\alpha)\tan\psi=\cot\phi/2$ and $\alpha=h/mc\lambda_0$, and the number of electrons recoiling in the direction ψ and crossing 1 cm^2 of the sphere can therefore be calculated. A formula for the angular distribution of the recoil electrons is given for any direction of polarization. This formula is applicable to Wilson photographs of recoil electrons.

1. Introduction

In two previous papers the writer has applied a corpuscular quantum theory to the scattering of both unpolarized[1] and polarized[2] x-rays. Since the first of these papers was written, C. T. R. Wilson[3] has published two papers on beta ray tracks obtained by using his expansion apparatus. Wilson's paper has been discussed by Compton and Hubbard,[4] who have pointed out the presence of tracks made by recoil electrons in Wilson's photographs and who have developed a formula for the spatial distribution of recoil electrons produced by unpolarized x-rays. In a paper published after the appearance of C. T. R. Wilson's papers F. W. Bubb[5] gives photographs of beta ray tracks produced by polarized x-rays. In Bubb's photographs tracks of recoil electrons are again evident. It therefore becomes necessary to develop a theoretical formula for the spatial distribution of recoil electrons produced by polarized x-rays. This formula is derived in the present paper.

2. Derivation of the Formula

Referring to Fig. 1 it has been shown in the writer's previous papers that when an x-ray corpuscle is scattered in the direction ϕ by an electron

[1] G. E. M. Jauncey, Phys. Rev. **22**, 233 (1923)
[2] G. E. M. Jauncey, Phys. Rev. **23**, 313 (1924)
[3] C. T. R. Wilson, Proc. Roy. Soc. **A104**, 1 and **A104**, 192
[4] Compton and Hubbard, Phys. Rev. **23**, 439 (April, 1924)
[5] F. W. Bubb, Phys. Rev. **23**, 137 (1924)

the latter recoils in a direction ψ which depends on ϕ according to the relation

$$(1+\alpha)\tan\psi=\cot\phi/2. \tag{1}$$

If there are Z electrons in the beam of x-rays, and if n_ϕ is the number of x-ray corpuscles scattered by the Z electrons in a direction ϕ and crossing unit area of a sphere of radius R in unit time, and if R is large in comparison with the dimensions of the volume containing the Z electrons and the center of the sphere is at the center of this volume before scattering takes place, then it can be shown by the method of the previous papers that when polarized x-rays are scattered

$$n_\phi=n_0G\frac{\sin^2\chi+\cos^2\chi\,\cos^2\phi-2F\cos 2\chi\sin^2\phi/2+(4F\cos^2\chi+F^2)\sin^4\phi/2}{(1+F\sin^2\phi/2)^4} \tag{2}$$

where $G=Ze^4/R^2m^2c^4$, n_0 is the number of x-ray corpuscles in the primary beam crossing unit area in unit time, ϕ is the angle of scattering, χ is the angle between the electric vector of the polarized primary x-rays and the plane containing the scattered and primary rays, and $F=2\alpha$ or $2\alpha+\alpha^2$

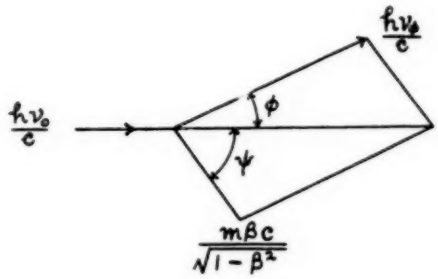

Fig. 1

where $\alpha=h/mc\lambda_0$ as in the previous papers. If N_ψ is the number of electrons recoiling in the direction ψ and crossing unit area of the sphere of radius R in unit time, then we have

$$N_\psi=n_\phi\,(\sin\phi/\sin\psi)\,(d\phi/d\psi). \tag{3}$$

Using Eq. (1) we obtain

$$(\sin\phi/\sin\psi)\,(d\phi/d\psi)=-4p/\{\cos^3\psi(1+p\tan^2\psi)^2\}, \tag{4}$$

where $p=(1+\alpha)^2$. The meaning of the minus sign is that ψ decreases as ϕ increases and we shall therefore drop this minus sign in the remainder of this paper. For brevity let us put $1+F=q$ and we have

$$n_\phi=\frac{n_0G(1+p\tan^2\psi)^2\{(q-p\tan^2\psi)^2+4pq\tan^2\psi\sin^2\chi\}}{(q+p\tan^2\psi)^4}. \tag{5}$$

The primary intensity $I_0=n_0h\nu_0$, where ν_0 is the frequency of the primary rays, and hence $n_0=I_0/h\nu_0$. Then multiplying Eqs. (4) and (5) we have

$$N_\psi = \frac{4GI_0p\left\{(q-p\tan^2\psi)^2+4pq\tan^2\psi\sin^2\chi\right\}}{h\nu_0\cos^3\psi(q+p\tan^2\psi)^4}. \tag{6}$$

This is the distribution formula for the recoil electrons produced by polarized x-rays. If this formula is averaged for all values of χ between 0 and $\pi/2$ we obtain the distribution formula for the case where unpolarized x-rays are scattered. This latter case has been discussed by Compton and Hubbard[4] using Compton's form of the quantum theory of scattering. My formula for unpolarized x-rays reduces to their formula when $F=2\alpha$.

It is worthy of notice that Eq. (6) becomes much simplified when $F=2\alpha+\alpha^2$ for then $q=(1+\alpha)^2=p$ and Eq. (6) reduces to

$$N_\psi = \frac{4GI_0\cos\psi\ (1-\sin^2 2\psi\cos^2\chi)}{h\nu_0(1+\alpha)^2}. \tag{7}$$

The electrons recoiling in a direction ψ have a kinetic energy

$$2h\nu_0\alpha\cos^2\psi/(1+2\alpha+\alpha^2\sin^2\psi), \tag{8}$$

and a momentum

$$2h\nu_0(1+\alpha)\cos\psi/c\ (1+2\alpha+\alpha^2\sin^2\psi), \tag{9}$$

as shown by Compton and Hubbard. It may be that the recoil electron does not possess sufficient energy to break free from the atom or if it does break free it may not possess sufficient energy to ionize the gas; in neither of these cases will recoil tracks be produced in the direction ψ. This aspect of the problem has been discussed by Compton and Hubbard' and as both the kinetic energy and the momentum of the recoil electron are independent of the direction of polarization χ the writer will not further discuss the matter here.

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DISSOCIATION OF HYDROGEN AND NITROGEN BY EXCITED MERCURY ATOMS

By O. S. Duffendack and K. T. Compton

Abstract

Dissociation of hydrogen in low-voltage arcs.—Arcs were maintained from a heated Mo filament to a Ni anode and the rate of dissociation was measured by the change of pressure produced as a result of oxidation of the atomic hydrogen by nearby copper oxide and of freezing out of the resulting water vapor. *In pure hydrogen* there is no dissociation, other than that due to the hot cathode, below a minimum voltage (13), but above the minimum arcing voltage (16) the rate was rather rapid, decreasing somewhat with increasing voltage to 65, the highest tried. *In mercury arcs*, however, the dissociation was at least 10 times more rapid in the 5 volt arc and 3 times more rapid in the 10 volt arc. This rapid dissociation is attributed to collisions of the "second kind" whereby the H_2 molecules receive energy for dissociation from excited Hg atoms in the $2p_2$ state.

Dissociation of nitrogen in low-voltage arcs.—Similar experiments with nitrogen, using pure Mg metal to absorb the dissociated nitrogen, showed a dissociating *effect of excited mercury atoms*, but only about one tenth as large as the effect in H_2. Reasons are given for believing that in N_2 the dissociation is produced by Hg atoms in the $2P$ state. This conclusion is supported by results obtained by others on the disappearance due to phosphorus vapor, but is apparently at variance with the similar effect of arsenic vapor. Clean-up *due to electron impacts, 17 to 100 volts*, is small to 25 volts, increases four fold to 70 volts, and 10 fold for voltages above 70. The effect is related to the intensity of the line spectrum. Above 70 volts, a very *active form of nitrogen* is produced, resulting in a "flare."

THE electrical "clean up," or disappearance of gases in an electrical discharge has been the subject of numerous investigations which have recently been critically reviewed by Dushman.[1] In general, the primary process appears to consist in putting the molecules of the gas into an active state, from which they disappear by some secondary process such as absorption or chemical action. In hydrogen the active state is generally the atomic state, produced by ionization,[2] although triatomic hydrogen may also be an important agent under certain conditions.[3] In nitrogen also the atomic and triatomic states appear to be active.[4] Probably in both cases the dissociation of the molecules is the primary step.

[1] Dushman, "High Vacuum" (1922)
[2] Langmuir, J. Am. Chem. Soc. **35,** 931 (1913)
[3] Wendt and Landauer, Am. Chem. Soc. **42,** 930, 1920; **44,** 510 (1922)
[4] Strutt, Proc. Roy. Soc. A **85,** 219 (1911); **86,** pp. 56 and 262, (1911); **87,** 179 (1912); **88,** 539 (1913); **91,** 303 (1915).
Lewis, Phys. Rev. **18,** 125 (1904)
Wendt and Grubb, Science **52,** 159 (1920)

The great importance of excited, or partially ionized, atoms in promoting spectroscopic and chemical actions in gases with which the excited atoms are mixed has recently been demonstrated by Franck and Cario.[5] Among other things they showed that hydrogen reacts with metallic oxides and is adsorbed by cooled glass walls if it is mixed with mercury vapor illuminated by the mercury resonance radiation 2536A. The mercury atoms which absorb the 2536 radiation are thereby changed from the normal $1S$ state to the "excited" $2p_2$ state whose potential energy is greater than that in the normal state by the equivalent of 4.9 volts. At collisions of these excited atoms with H_2 molecules, this energy may be transferred and lead to dissociation of molecules into atoms of hydrogen, a process requiring about 3.6 volts energy, according to Langmuir.[6]

This process of dissociation of molecules by action of excited atoms should occur in low voltage arcs, under suitable conditions. If, for instance, a low voltage arc is maintained in mercury vapor in the presence of hydrogen, we should expect a very rapid clean-up of the hydrogen. This is because the condition necessary to the maintenance of a low voltage arc is a large concentration of excited atoms, which is also a condition for rapid dissociation of the admixed hydrogen. We therefore carried out the following experiments to compare the importance of this method of dissociation with that produced by a hot filament alone or by direct electron impacts with the gas molecules.

Hydrogen

The apparatus consisted simply of a glass bulb containing a short 15 mil filament to serve as the cathode source of electrons, and a nickel disk anode placed about 1 cm from the filament. Below these electrodes was a glass tray containing CuO placed in a favorable position for the oxidation of any atomic hydrogen formed near the electrodes. Traps cooled by liquid air or by solid CO_2 were provided to eliminate the water vapor as rapidly as possible, in order to preserve the hot filament. The rate of disappearance of the hydrogen was found by measuring the pressures, with a McLeod gauge, at intervals of 1 minute.

Mercury could be introduced into one end of the bulb, and any desired vapor pressure obtained by regulating the temperature of the heating

[5] Franck, Zeits. f. Phys. **9**, 259, (1922);
Cario, ibid., **10**, p. 185 (1922);
Franck and Cario, ibid., **11**, 161 (1922);
Cario and Franck, ibid. **17**, 202, (1923)

[6] Langmuir, J. Am. Chem. Soc. **34**, 860 (1912)

oven in which the bulb was placed. In order to maintain low voltage mercury arcs at or near the resonance potential 4.9 volts, a fairly high vapor density is required; this was obtained by operating the oven at about 235°C. Arcs could be maintained near the ionizing potential 10.4 volts by reducing the oven temperature to about 150°C. Hydrogen arcs could be maintained at any voltage above the ionizing potential of hydrogen, about 16 volts, by reducing to room temperature or by removing the mercury entirely. The rate of disappearance of hydrogen was studied under these various conditions.

Langmuir[6] first showed that hydrogen molecules are dissociated by contact with a hot filament and that the fraction of those striking the filament which are dissociated increases rapidly as the filament tempera-

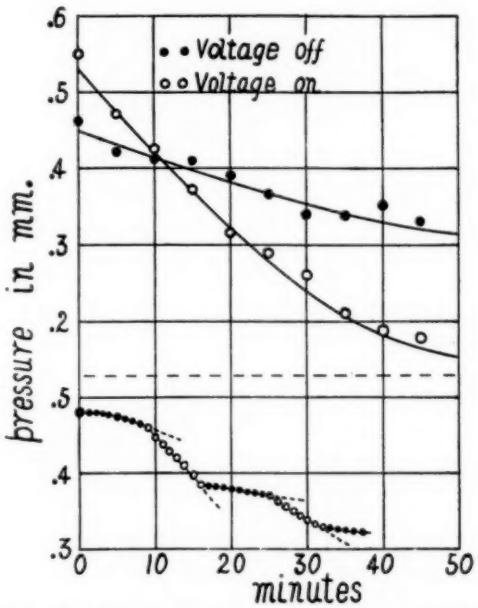

Fig. 1. Decrease of hydrogen pressure with time with 5.3 volt discharge in the presence of mercury vapor. With the voltage off the decrease is due only to the dissociating action of the hot filament. With the voltage on, the dissociation by excited atoms is added to that by the filament. At this voltage there is no dissociation by electron impacts in pure hydrogen. Discharge current about 14 milliamperes.

ture is raised. Dissociation from this source was always present in our experiments, superimposed on the dissociation produced electrically. In order to reduce this thermal dissociation we used molybdenum filaments, thereby securing the desired electron emission at a lower filament temperature than if tungsten had been used. This greatly facilitated the interpretation of the results, for, by maintaining the filament temper-

ature as constant as possible, we could compare rates of clean-up due to different methods of electrical action.

Fig. 1 illustrates the rate of clean-up of hydrogen in a low voltage mercury arc, in which the concentration of excited mercury atoms is known to be large. The dots represent observations with the voltage (and hence the arc) off, so that the slope of the curve gives the rate of clean-up due to thermal dissociation by the filament. The circles are observations at the same filament temperature, but with the voltage (5.3 volts) applied. The slope of this curve gives the rate of clean-up due to the combined action of the hot filament and of the excited mercury atoms.

Fig. 2 illustrates the rate of clean-up in a mercury arc near the ionizing potential. The effect of the arc is not as large, relatively to that due to the hot filament, as in the low voltage arc.

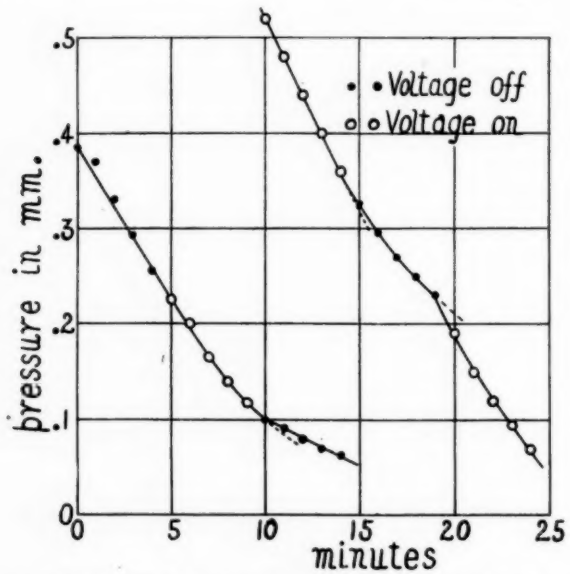

Fig. 2. Rate of clean-up of hydrogen with and without the 10.4 volt arc in the presence of mercury vapor. Discharge current about 14 milliamperes.

Similar experiments in pure hydrogen, with arcs at various voltages and conditions similar to those in Figs. 1 and 2 except for the absence of mercury vapor, showed a much diminished rate of electrical clean-up. This is shown by Fig. 3, in which the abscissas are voltages at which arcs were maintained and the ordinates are the ratios of the rates of clean-up with and without the arc. In other words, the ratio $R' = (\text{thermal} + \text{electrical clean-up})/(\text{thermal clean-up})$. The rates of clean-up

R due to electrical effects alone are found by subtracting 1 from the values of R' and are shown to the right of the diagram, reduced to mm per minute per milli-ampere of arc current.

Owing to action of water-vapor on the filament it was impossible to maintain all conditions as constant as might be desired, and consequently the observed rates of clean-up under initially similar conditions varied somewhat in different runs. The average rate in the low voltage mercury arc is shown by L and that for the higher voltage are by M.

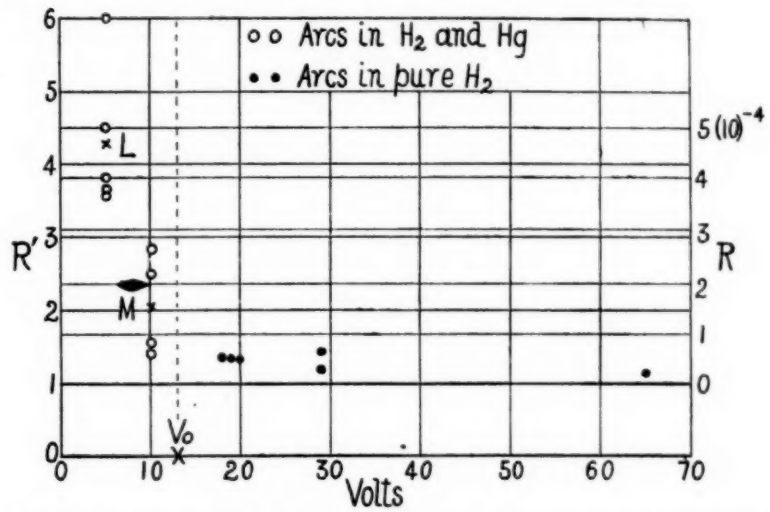

Fig. 3. R' is the ratio of the rate of clean-up from filament and electric discharge together to that from the filament alone. R is the rate of clean-up due to the discharge alone in millimeters per minute per microampere. At voltages below V_0 there is no clean-up due to the discharge in pure hydrogen. L shows the average rate of clean-up in a 5.3 volt arc and M in a 10.4 volt arc in the presence of mercury vapor.

The general conclusion to be drawn from the experiments is clear: *The rate of clean-up of hydrogen in a 5.3 volt mercury arc is about 10 times more rapid and in a 10.4 volt mercury arc about 3 times more rapid than in a pure hydrogen arc at any voltage up to 65 volts* (the highest tried). This is in spite of the unfavorable effect of the mercury vapor in slowing up diffusion of atomic hydrogen to the absorbing CuO tray and facilitating recombination.

That this effect of the presence of mercury can in no way be attributed to any effect of electron impacts against hydrogen atoms is certain, since Hughes has shown by careful experiments[7] that there is no detectable clean-up due to electron impacts in hydrogen at any voltage below 13

[7] Hughes, Phil. Mag. **41**, 778 (1921)

volts. We therefore conclude that dissociation of hydrogen at collisions of the "second kind" with mercury atoms which have been excited by electron impacts is a much more efficient process than dissociation by direct electron impacts against hydrogen atoms. In fact, recent experiments by Smyth[8] and by Keussler,[9] suggest that the dissociation which is observed even in pure hydrogen between 13 and 20 volts arises, at least in part, from collisions of the second kind with excited hydrogen molecules. The relatively high dissociating efficiency of the excited mercury atoms is due to the fact that their energy in the excited state is not greatly in excess of the amount necessary to dissociate the hydrogen molecules, for Klein and Rosseland have shown[10] that the probability of a collision of the second kind is greatest if the energy of the excited atom is just greater than the amount necessary to produce the effect.

Nitrogen

We next investigated nitrogen by the same methods in order to determine whether nitrogen molecules are dissociated upon impact with excited mercury atoms as was the case with hydrogen. It has long been known that nitrogen is activated by an electric discharge. One form was found to be responsible for the after glow when a discharge is passed through nitrogen, and its spectrum was investigated by E. P. Lewis and reported in a series of papers.[11] The chemical properties of this form were studied by Strutt and Fowler and their pupils.[4] Strutt reached the conclusion that this form was due to the dissociation of the nitrogen by a condensed, disruptive discharge. An activation similar to this in many respects was reported by one of us[12] as being formed in a low voltage arc in nitrogen at voltages of 70 volts or more. A different type of activation has recently been discovered by Wendt and Grubb[4] in the corona discharge. They attribute the activation in this case to the formation of an ozone form with a larger molecule than N_2.

The nitrogen used was obtained by dropping a dilute solution of bromine into a solution of ammonia. The gas was purified by passing it through a saturated solution of potassium hydroxide, over which it was held for some time, then holding it in a tube containing heated copper turnings, passing it over phosphorous pentoxide, and finally through a tube immersed in liquid air. Turnings of pure magnesium metal were

[8] Smyth, Proc. Roy. Soc. A (in print)
[9] Keussler, Zeits. f. Phys. **14,** 19 (1923)
[10] Klein and Rosseland, Zeit. f. Phys. **4,** 46 (1921)
[11] Lewis, Ann. d. Physik, **2,** 466 (1900); Phys. Rev. **18,** 125 (1904); Astrophys. Jour. **20,** 49 (1904); Phil. Mag. **25,** 826 (1913)
[12] Duffendack, Phys. Rev. **20,** 665 (1922)

placed in the discharge tube along with carefully purified mercury. Runs were also made with no mercury in the tube, and, likewise, with no magnesium. The discharge tube was thoroughly baked out at 300°C to 350°C and the mercury further purified at the same time by allowing it to distill into a condenser and run back into the tube.

The experimental procedure and interpretation of results was simpler than in the work with hydrogen owing to the absence of clean-up from the filament alone. The heat of dissociation of N_2 is unknown, but Langmuir has shown[2,6] that it is not appreciably dissociated by contact with tungsten at any temperature at which a filament can be run. The only clean-up observed is due to reaction of nitrogen with tungsten vapor and this was too small to be detected in the presence of the electrical clean-up in our experiments.

The rate of disappearance of nitrogen was much less than that of hydrogen in all cases, less even than the rate of electrical clean-up in pure hydrogen. Under similar conditions the rate for nitrogen per milli-ampere arc current per minute was about one tenth that of hydrogen, up to about 70 volts potential difference across the arc, and reached a value about half that of hydrogen for voltages higher than this. While it is possible that the copper oxide in the hydrogen tube facilitated the clean-up of hydrogen to a greater extent than the agents employed with nitrogen, on the other hand, the tungsten filaments used are known to be effective in removing atomic nitrogen and did not combine with the hydrogen.

When the arc was maintained in mercury vapor below the ionizing potential of nitrogen, the rate of disappearance of the gas was greater than when the arc was maintained in nitrogen in the absence of mercury vapor at voltages of 17 to 25 volts, as shown by Fig. 4. At higher voltages, 40 to 60 volts, the presence of mercury vapor at a pressure equal to its vapor tension at room temperature, about 0.0015 mm, apparently had little effect on the rate of clean-up. Hughes,[7] however, observed an increase in the rate when mercury vapor from his pump was permitted to enter his tube. He suggests that possibly the mercury vapor helps to "fix" the atomic nitrogen as it is formed. As the rate of disappearance of nitrogen is much less than that of hydrogen in the presence of excited mercury, it does not seem likely that the effect can be due to the action of mercury atoms in the excited state corresponding to the absorption of resonance radiation of wave-length 2536, i.e. to atoms having their valence electrons displaced to the $2p_2$ orbit. In this state the mercury atom has a potential energy of 4.9 equivalent volts, and, if this were sufficient to dissociate a nitrogen molecule, as

much dissociation would be effected in nitrogen as in hydrogen at the same pressure unless there is something in the mechanism of the process which makes the transfer of this energy less probable at impact with a nitrogen molecule. It is probable that, if the heat of dissociation were less than 4.9 volts, although known to be greater than 3.6 volts (hydrogen), the clean-up would have been *faster* than in hydrogen owing to the greater probability of a collision of the second kind.

A more plausible explanation is that the effect observed in nitrogen is due to mercury atoms in a higher state of excitation, possibly those with electrons displaced to the $2P$ orbit possessing a potential energy of

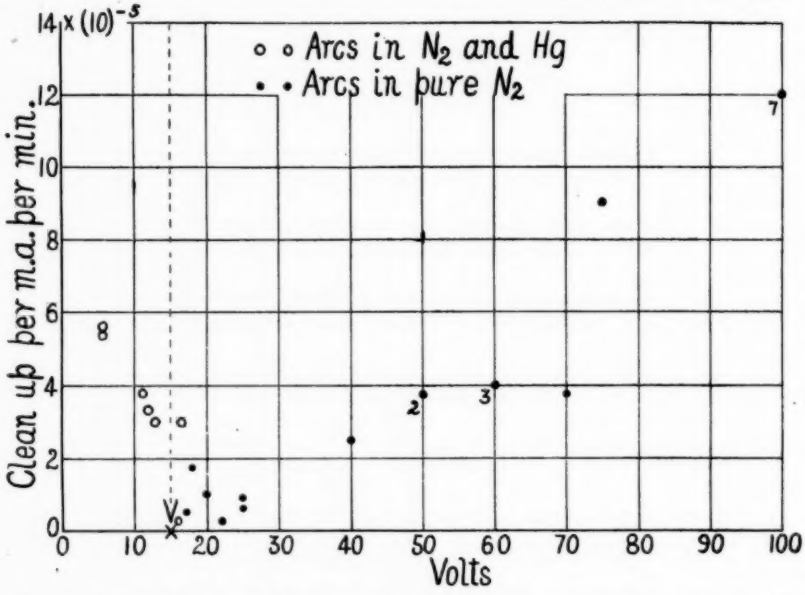

Fig. 4. Rate of clean-up of nitrogen in discharges at various voltages with and without the presence of mercury vapor. V_0 is the voltage below which no clean-up is observed in pure nitrogen. Points marked 2, 3, 7 are mean positions of 2, 3 and 7 sets of observations.

6.6 volts. As there are fewer of these present in the mercury arc, this would account for the lesser effect in nitrogen than in hydrogen. This would mean that the heat of dissociation of nitrogen is greater than 4.9 volts and less than 6.6 volts. While its value is not known, estimates put upon it have usually been less than 6.6 volts.

Possible Action of Phosphorous Vapor

In connection with the disappearance of nitrogen from a tube containing an electric arc in mercury vapor, it might be well to discuss a similar disappearance from a discharge tube containing phosphorous

vapor reported by the Research Staff of the General Electric Company, London.[13] In their investigation they were able to get a lower final pressure of gas when phosphorous vapor was present than when it was not. In fact, the final pressure was limited by the disappearance of the phosphorous vapor and the consequent stopping of the discharge. They point out that the action of the phosphorous cannot be chemical and that ionization is essential, and suggest that the phosphorous is deposited upon the walls of the tube as red phosphorous and covers up the film of the gas adhering there and forms a new surface on which the gas can be absorbed.

By analogy with the cases just discussed, we might expect that dissociation of N_2 by excited phosphorous atoms or molecules is an important feature in the disappearance of nitrogen under these conditions. Phosphorous has a resonance potential of 5.8 volts, being the next higher than mercury, 4.9 volts, among all the vapors thus far examined. If, therefore, the heat of dissociation lies between 4.9 and 5.8 volts (113,000 and 133,000 calories per gm. mol.), we would expect a rapid dissociation. Other gases, such as the permanent and inert gases, have resonance potentials so much higher that they would be much less efficient in giving collisions of the second kind. These limits place the heat of dissociation of nitrogen about where it is expected to be, but should not be accepted without further tests of the validity of the suggestion here offered.

Apparently at variance with this suggestion is the fact that arsenic vapor, with a resonance potential of only 4.7 volts, acts as does phosphorous. In these vapors, in a discharge, the molecular state is certainly varied and there are probably other and higher resonance potentials than those thus far discovered. Further experiments along these lines, under conditions in which the voltage and other conditions can be accurately regulated, are being undertaken.

Activation of Nitrogen in a Low Voltage Arc

Below the ionizing potential in pure nitrogen there is no detectable clean-up due to electron impacts.[14] At voltages a little above the ionizing potential, 17 to 25 volts, the rate of disappearance of the gas is very small and only about one fourth that at voltages less than the ionizing potential in the presence of excited mercury. The rate increases about four fold for voltages from 40 to 70 volts and about ten fold for voltages above 70 volts. This fits in with the observations of one of us,[12] and more recently, by D. C. Duncan[15] that the nitrogen line spectrum does

[13] Phil. Mag. **40,** 585 (1920); **41,** 685 (1921); **42,** 227 (1921); **43,** 914, (1922)

[14] Hughes, loc. cit.[7]

[15] To be published.

not appear in weak arcs below about 70 volts. It would seem, then, that the disappearance of the gas in this case is connected with the dissociation of the molecule and that the molecule is usually ionized without dissociation at low voltages.

At voltages above 70 a very active form of nitrogen appears, that referred to in the first part of the discussion. The conductivity of the arc increases enormously and the arc turns into a brilliant "flare." The action on the electrodes was so vigorous that this discharge could not be maintained long enough in the tube used to determine whether the gas is used up in the discharge or not. The light supporting wires for the anode were quickly melted and then the discharge played on a stub of 100 mil nickel wire that had held the support wires. The end of this stub became white hot and quickly fused, sending little globules of nickel sputtering to the walls of the tube. The current through the arc was about four amperes at this time. The action of the nickel seemed to be more as a catalytic agent than as a constituent in a chemical reaction. There was little evidence for the formation of a compound with the nickel except for a slight tarnishing of the surface. The amount of nickel taken off the stub was more than sufficient to use up all the nitrogen in the apparatus. The globules of nickel that spattered the walls of the tube were also somewhat tarnished but little changed. After maintaining this discharge for about a minute, the pressure in the apparatus showed a gain instead of a loss. This increase was probably due to an evolution of gas from the electrodes, especially the anode, as a consequence of its heating and fusing. The spectrum of this "flare" shows the lines of the metals of the electrodes as well as various lines and bands of nitrogen. The magnesium present in the tube was apparently little affected by this discharge, while that used with mercury in the work previously described became coated with a black powdery substance. This difference may have been due to the fact that the magnesium was heated in the mercury runs but not in the runs without mercury.

These experiments were carried out at the University of Michigan during the summer of 1923.

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THE BALMER ABSORPTION SERIES OF HYDROGEN

By E. O. Hulburt

Abstract

Laboratory method of obtaining ten Balmer lines in absorption.—Condensed discharges were passed through an under water spark in series with a tube, 75 cm in length, of moist hydrogen at a pressure of 0.5 mm of mercury. The light from the spark traversed the glowing hydrogen, and the Balmer lines appeared as narrow dark absorption lines against the continuous spectrum of the spark, the absorption series extending to the tenth Balmer line. By adjusting the relative intensities of the light of the spark and of the glowing hydrogen, spectra were obtained with Hα and Hβ either bright or dark while the higher members of the series remained always dark. Spectra were produced showing Hβ and Hγ dark but with a bright central core. These phenomena are well known in stellar spectra, to which reference is made.

IN an investigation of unusual interest R. W. Wood[1] by means of long vacuum tubes has extended the Balmer emission spectrum of hydrogen to the twentieth member. Recently Whiddington[2] has described a different method of accomplishing this. The Balmer absorption spectrum, on the other hand, although well known in stellar spectra to the thirty-second line, has never been produced in the laboratory beyond Hβ. Several years ago Ladenburg and Loria[3] succeeded in reversing Hα and Hβ by passing the light from a short tube filled with hydrogen at considerable pressure through a long tube of hydrogen at a low pressure. The two tubes were connected in series and were excited by condensed discharges. The absorption lines appeared as fine dark lines against the broadened emission lines of Hα and Hβ. Attempts[4] which have been made to extend the absorption to the other members of the series have met with but little success. It is not obvious why these attempts have failed. In the present experiments by a slight variation of Ladenburg's method the absorption series has been extended to the seventh line using dry hydrogen, and to the tenth line using moist hydrogen.

The experimental arrangements are indicated in Fig. 1. The 30 kv, 1 kw, transformer P charged the condenser C, of capacity 0.015 microfarads, which discharged through the quenched gap Q, the long hydrogen

[1] R. W. Wood, Roy. Soc. Proc. **97,** 455 (1920); Phil. Mag. **44,** 538 (1922)
[2] Whiddington, Phil. Mag. **46,** 605 (1923)
[3] Ladenburg and Loria, Verh. Deut. Phys. Ges. **10,** 858 (1908)
[4] Foote and Mohler, Origin of Spectra, p. 98 (1922)

tube T, the underwater spark S, and the ammeter A. The auxiliary gap Q was necessary to insure abrupt discharges. The tube was of glass, 75 cm in length between the electrodes and 2.3 cm in diameter, with aluminum electrodes. The electrodes of the underwater spark S were of phosphor bronze, for this metal disintegrated slowly and the spark emitted a very intense continuous spectrum in the region under investigation. The light from the spark S, rendered slowly converging by the lens L, passed through the glowing hydrogen tube and came to a focus on the slit H of a concave grating spectrograph of dispersion 8.8 A per mm.

Spectrograms taken with dry hydrogen in the tube showed the Balmer lines as fine dark absorption lines against the continuous emission from the under water spark. The series was recorded to the seventh Balmer

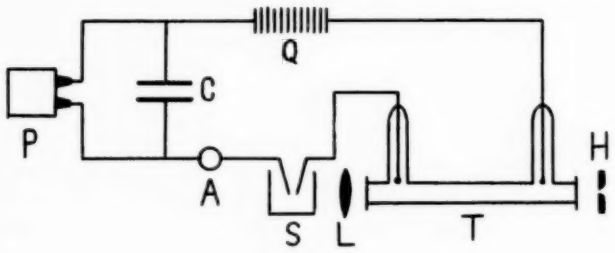

Fig. 1

line. To extend the series farther, use was made of the fact, long known,[5] that the presence of a slight amount of water vapor in a hydrogen discharge tube suppresses the secondary spectrum and intensifies the Balmer spectrum. Moist hydrogen was admitted into the tube through a slender capillary tube in the manner described by Wood.[1] The best absorption spectrograms, which recorded the reversed Balmer lines to H_{10}, shown in *a*, Plate 1, were obtained with a pressure of about 0.6 mm of mercury (dark space 5 mm) and with a current of about five amperes through the tube. The time of exposure was five minutes. In the emission spectrum of the hydrogen tube taken with an exposure of 15 minutes, the series lines could be traced only to H_6. With a five minute exposure the emission spectrum barely included H_4. It is evident that under these conditions the absorption brings out many more lines than the emission. Apparently the stimulated atoms are very effective absorbers for the Balmer radiations. This is in keeping with the theory of absorption discussed mathematically by Lamb[6] who pointed out that a single resonator can exert enormous checking power upon advancing

[5] Merton and Nicholson, Roy. Soc. Proc. **96,** 116 (1919)
[6] Lamb, Cambridge Phil. Soc. Trans. **18** (1900)

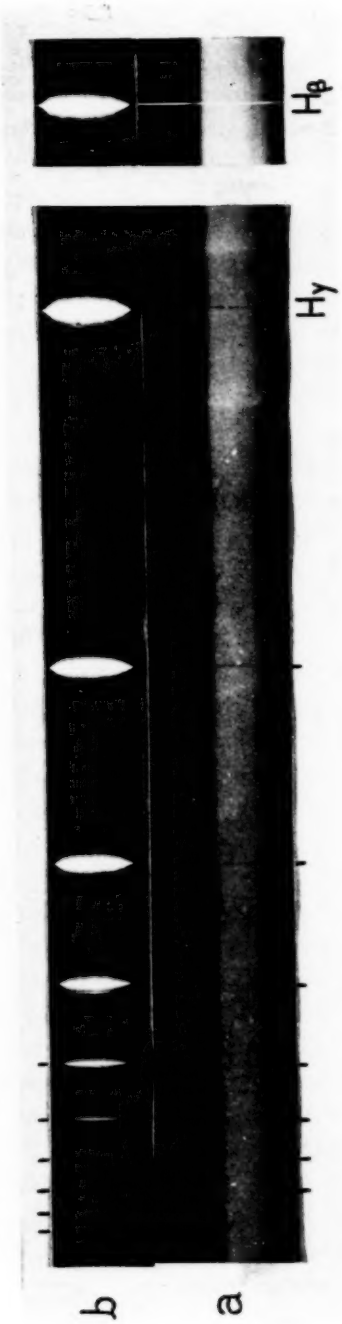

Plate 1

(a) Absorptions spectrogram, showing reversed Balmer lines to H_{10}.
(b) Emission spectrum of hydrogen tube, showing lines to H_{12}.

radiation of a frequency very nearly, but not quite, that of its own period. A similar instance is the high opacity of mercury vapor for the mercury line 2536.[7]

In an attempt to extend the absorption series farther into the ultra-violet a long tube was made with a straight portion 40 cm in length, 7 mm internal diameter, and with long side branches leading to the electrodes, such as described by Wood.[1] The present experiment necessitated a departure from Wood's method in that powerful condensed discharges were required instead of sixty cycle currents. With the condensed discharges the Balmer emission lines from the long tube became somewhat diffuse because of the Stark broadening, and with an exposure of ten minutes only twelve lines could be traced under the best conditions which were obtained. Spectrogram *b* of the Plate illustrates this. When the absorption was attempted with this tube the Balmer lines remained bright because the small bore of the tube effectually prevented the passage of the continuous spectrum in sufficient intensity. To overcome this difficulty a long tube was constructed with an internal diameter of 2.3 cm instead of 7 mm. Although various types of electrical excitation were tried, all attempts failed to bring this tube completely into the "black stage," as Wood has called it, i.e., the Balmer lines strong and the secondary weak, so that only eight Balmer absorption lines were recorded. The walls of the tube were perhaps not close enough to all parts of the gas to catalyze effectively the molecules to the atomic state. A more intense electrical excitation of some sort might have been necessary.

It is seen that all the Balmer lines of spectrogram *a* were reversed except Hβ which was bright. A visual observation at the time showed that Hα also was bright against the continuous background. Decreasing the brilliancy of the glowing hydrogen by admitting a small amount of hydrogen into the tube, brought out Hβ dark, instead of bright, against the continuous spectrum. Increasing the brightness of the glowing hydrogen gave a spectrum with Hα, Hβ and Hγ appearing as emission lines and the other lines as absorption lines. The explanation of these observations is to be found in the conclusions of Ladenburg[8] as a result of an investigation of the absorption of Hα. By means of Nicol prisms the relative brightness of Hα and the back-ground illumination were varied at will. It was established that in the region of wave-lengths covered by the Balmer line the light from the background was completely absorbed, and that the light which did appear in this region was emitted from the long column of stimulated hydrogen. Therefore the appearance

[7] Wood, Researches in Physical Optics, Part 1.

[8] Ladenburg, Verh. Deut. Phys. Ges. **12,** 54 and 549 (1910)

of the Balmer line as bright or dark depended primarily upon the relative brightness of the two sources.

The spectrograms of the present experiments find interesting counterparts in certain stellar spectra,[9] some stars giving spectra with all the hydrogen lines bright, others with all the hydrogen lines dark, and still others, such γ *Argus* and η *Tauri*, with the earlier members of the Balmer series bright and the shorter wave-length hydrogen lines dark.

Spectrograms were obtained in which Hα and Hβ were bright, Hγ was dark with a bright centre, and the other lines were entirely dark. The complex character of Hγ, although unmistakable on the negative, did not lend itself successfully to an enlarged reproduction, for the extreme width of Hγ on the plate was only about 0.2 mm and the edges of the bright core were hazy. Other spectrograms under slightly different conditions showed Hα bright, Hβ dark with a bright centre and the remaining lines dark. That the dark absorption line may have a bright core was observed by Ladenburg[8] in the case of Hα who proved that the dark edges of the line resulted from an absorption by the long layer of glowing hydrogen, greater than the emission, whereas in the bright centre the reverse was true. Again these facts are well known in the spectra of the stars. For instance, the quasi-permanent stars μ *Centauri* and *J Velorum*[10] at one time exhibited Hβ and Hγ as absorption lines with well marked emission centres. In the spectrum of II *Camelopardalis*[11] Hγ and at least seven of the Balmer lines which follow it are dark with bright centres of emission.

In this connection we may mention a complexity of certain stellar hydrogen lines which as yet has not been imitated in the laboratory, as for example Hγ, Hδ and Hϵ in the spectrum of φ *Persei* and Hβ and Hγ in the spectrum of β *Monocerotis*.[12] These lines are broad dark absorption bands with a narrow bright centre band which in turn is divided by a yet narrower line of absorption. Such a complex line would perhaps result if the following conditions existed: (1) a hot dense stellar interior yielding a continuous spectrum, (2) an absorbing envelope of hydrogen, relatively dense, giving a wide absorption line, (3) above this a region of less pressure and greater ionization which emitted somewhat narrower lines, and (4) above this yet another region of cooler, more rarified hydrogen causing the still narrower center of absorption.

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UNIVERSITY OF IOWA,
January 5, 1924

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THE CURRENT-VOLTAGE RELATION IN THE CORONA

By C. S. Fazel and S. R. Parsons

Abstract

Current-voltage relation for the corona discharge between a cylinder and a coaxial wire.—*An equation* is derived on the basis of a small region (of radius a) of intense ionization around the wire, outside of which the current is carried only by ions of one sign, whose space density ρ is constant. At the boundary between the two regions the field is assumed to be the minimum field required to start the corona, and the field beyond is taken to be the sum of the electrostatic field and that due to the space charge. Putting in the experimental result that a is a linear function of the applied voltage V, the equation for the current i is put in the form $i = cV(V - V_0)/(V_1 - V)$, when c is a constant proportional to the mobility. *Measurements* with a brass tube 17.8 cm long and 4.75 cm inside diameter, through which a slow stream of dried or moist air was passed, show good agreement with the above equation as far as variation with V is concerned, for voltages from 4.7 kv to 8.8 kv, for temperatures from 290°K to 417°K and for moisture content up to 44 percent. The theory is evidently imperfect, however, as the absolute values of the mobilities come out from 2 to 4 times the values obtained in small fields although the variation with absolute temperature is correct.

THE relation between current and voltage in the corona discharge between a hollow cylinder and a wire placed at its axis has been studied by Almy,[1] who gave an empirical equation, and by Schaffers[2] and Townsend,[3] who have derived theoretical equations. Both Schaffers and Townsend make use of the fact that ionization takes place principally within a small region surrounding the wire, and obtain relations involving the mobilities of the ions. Schaffers' equation, however, yields mobilities as high as one hundred times as great as those obtained by direct measurement, and Townsend's equation is not in a convenient form, because it expresses the current as an implicit function of the voltage. These difficulties may be removed, as will be shown below, by a treatment of the problem somewhat similar to that of Schaffers, but taking account of the fact that the region of intense ionization expands as the voltage is increased.

From the fact that the ionization takes place almost wholly within a certain short distance of the wire, it follows that beyond this distance,

[1] Almy, J. C., Am. Jour. of Sci. (4), **12**, 175, 1901
[2] Schaffers, V., Phys. Zeits. **15**, 405, 1914
[3] Townsend, J. S., Phil. Mag. (6), **28**, 83, 1914

the current is carried by ions all of the same sign, constituting a space charge. For example, if the wire is positive, both positive and negative ions are formed in the region immediately surrounding the wire, but since the negative ions move in toward the wire, and the positive ions move outward, the space outside of the ionizing region contains positive ions only. Our attention will be directed principally to this region of space charge, i.e., outside of the region of intense ionization. If the field intensity is not too great, nor the gas pressure too low, the motion of the ions that carry the current is somewhat similar to the fall of very small particles through a viscous fluid, and their velocity is proportional to the intensity of the field.

The current density at any point is given by

$$j = \rho u = \rho k X$$

where ρ = volume density of charge; u = velocity of ions; k = mobility of ions; X = electric field. Hence the total current per unit length of wire is

$$i = 2\pi r \rho k X. \tag{1}$$

If the current is small, the field X will be nearly that of the electrostatic case, namely,

$$X = -\frac{V}{r \log (R/r_0)}$$

where R = radius of cylinder; r_0 = radius of wire.

This expression, however, will not be used as we are confining our attention to the space outside of the region of intense ionization. The visible glow gives an indication of the extent of the ionization, and photographs[4] of the discharge show fairly definite limits to the luminous region. Let a = radius of the cylindrical surface which we shall regard as the inner boundary of the region in which Eq. (1) is applicable. Then the potential difference between the cylinder and the wire will be nearly equal to that between the cylinder and the surface $r = a$, since a is only a little greater than the radius r_0 and the ionizing potential of the gas is only a few volts; and to a fair approximation we may write

$$X = -\frac{V}{r \log (R/a)} \tag{2}$$

Combining this relation with Eq. (1),

$$i = -2\pi\rho k \frac{V}{\log (R/a)}. \tag{3}$$

This equation is not in convenient form for use, because the density of charge ρ depends upon the potential difference; but by means of Poisson's equation another relation may be obtained, which makes possible the elimination of ρ.

[4] Peek, F. W., Jr., Dielectric Phenomena in High Voltage Engineering, p. 76.

Assuming the dielectric constant of the gas to be unity, Poisson's equation for this case may be put in the form

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\phi}{dr}\right)=-4\pi\rho$$

where ϕ = potential. This equation is easily integrated, for it will be seen from Eq. (3), in which r does not appear, that ρ cannot be a function of the radius. Indeed, for a particular potential difference V, with the corresponding current i, all the other quantities in Eq. (3) are constants, and therefore ρ must be constant, or the charge density is uniform throughout the region considered. The first integration of Poisson's equation yields

$$r\,d\phi/dr=-2\pi\rho r^2+C_1.$$

Now a certain definite voltage V_0 is required to start the corona current, and corresponding to this starting voltage is a definite field intensity X_0 at the wire, which causes the air to break down, and may be called the "starting" field. When the voltage is raised above the starting value it seems reasonable to believe that the surface at which the "starting" field is maintained is no longer the wire but will be located on a cylinder of radius a in the gas. But this surface is the inner boundary of the region to which we are applying Poisson's equation, and therefore the constant of integration C_1 may be determined from the condition that $d\phi/dr=-X_0$ when $r=a$. But by comparison with Eq. (2),

$$X_0=-\frac{V_0}{a\log(R/a)}.$$

This condition gives

$$C_1=2\pi\rho\,a^2+\frac{V_0}{\log(R/a)}.$$

But both ρ and a are small, so that the first term of the right-hand member of the equation may be neglected,[5] and we may write

$$r\frac{d\phi}{dr}=-2\pi\rho\,r^2+\frac{V_0}{\log(R/a)}.$$

Integrating this equation between the limits a and R,

$$V=-\pi\rho(R^2-a^2)+V_0.$$

Now a^2 may be neglected in comparison with R^2, giving

$$V=-\pi\rho\,R^2+V_0. \tag{4}$$

Substituting in this equation the value of ρ from Equation (3),

$$i=\frac{2kV(V-V_0)}{R^2\log(R/a)}. \tag{5}$$

[5] In an extreme case, where $i=25\ (10^{-6})$ amp. per cm, and $V=8.5$ kv, ρ is about 2.2 e.s., and a is always a small fraction of a centimeter, while X_0 (which is less than the last term of this equation) is about 39 kv per cm, or 130 e.s.u.

It remains to express the distance a in terms of other quantities. The surface $r=a$ must be closely related with the region from which light is emitted in the corona discharge. From the photographs mentioned above it is seen that the apparent size of the wire increases as a linear function of the applied voltage. It thus seems possible that a may be expressed as such a function:

$$a=\alpha+\beta V.$$

This will introduce an empirical feature into the resulting equation but it will be possible to obtain the values of α and β from the experimental curves. When the applied potential is just sufficient to cause the corona to start, the ionization begins at the surface of the wire, so that

$$a=r_0 \qquad \text{when} \quad V=V_0,$$

and the equation may be put in the form

$$a=r_0+\beta(V-V_0).$$

Then

$$\log \frac{R}{a}=\log \frac{R}{r_0}-\log \left[1+\frac{\beta}{r_0}(V-V_0)\right].$$

The quantity $(\beta/r_0)(V-V_0)$ has usually the magnitude of a few tenths, so that the $\log [1+(\beta)r_0)(V-V_0)=\beta/r_0(V-V_0)$, approximately. Eq. (5) may now be written

$$i=\frac{2kV(V-V_0)}{R^2\left[\log \frac{R}{r_0}-\frac{\beta}{r_0}(V-V_0)\right]}=\frac{2kV\ (V-V_0)}{R^2\frac{\beta}{r_0}\left[\frac{r_0}{\beta}\log \frac{R}{r_0}+V_0-V\right]}$$

Collecting constant quantities, and letting $2kr_0/R^2\beta=C$ and

$$\frac{r_0}{\beta}\log \frac{R}{r_0}+V_0=V_1$$

the equation may be put in the final form

$$i=\frac{CV\ (V-V_0)}{V_1-V}. \tag{6}$$

The accuracy of Eq. (6) was tested by comparison with available curves of the current-voltage relation,[6] and found to be very satisfactory. As a further check, a number of experimental determinations of this relation were made, and the range of conditions was extended by using different temperatures and different humidities of the air. Alternating current was used, and the starting potential was computed in each case from the curve of the current-voltage relation.

Experimental Method

The corona tube was of brass, 17.8 cm long and 4.75 cm in diameter inside, with a central copper wire, No. 29 B. & S. gauge, held in place by

[6] Univ. of Ill., Eng. Exp. Station, Bulletin No. 114, p. 37, 38.

glass plates attached to the ends of the tube by bakelite cement. The voltage applied to the tube was read by means of a potential transformer and an ordinary voltmeter, and the corona current was measured by a thermal cross in an evacuated tube, connected to a sensitive galvanometer. In order to remove the ozone, which forms rapidly, and to insure that the current was measured in air rather than in a mixture of air and ozone, a steady slow stream of air was maintained through the apparatus, passing through sulphuric acid and over phosphorus pentoxide before entering the corona tube, and escaping through a tube whose mouth was just below the surface of water, in order that the rate of bubbling might be used as a rough indication of the speed of flow. Possible diffusion of water vapor from the water back into the apparatus was prevented by the insertion of a second tube of pentoxide.

For a study of the effects of humidity, the air, instead of passing through the drying tubes, was forced in succession through two bottles of water, each provided with an electric heater, and was led around the drying tube at the outlet.

For the control of temperature, the corona tube was placed in a large wooden box, lined with asbestos, and provided with electric heaters. This box also contained brass cylinders about 20 cm long and 7.5 cm in diameter, through which the air passed. In order to insure complete drying, or complete saturation, the flow must evidently be quite slow, and at such speeds it is not practicable to heat the air by passing it through *small* heated tubes. The speeds were far below the critical velocity, and in the viscous flow thus obtained, the transfer of heat from the tube to the air is very slight. The cylinders gave satisfactory heating by providing reservoirs in which the air remained for some time, since they were of sufficient size to allow free convection currents. The controlling temperature measurements were made with copper-constantan thermocouples, two on the corona tube itself, and one in the air stream near the entrance to the tube. Other junctions were placed in the reservoirs of both dry and moist air streams, and in the second water bottle, which was outside of the wooden box, in a smaller box of its own, heavily lagged with hair felt.

It was found by trial that irregularities in readings could be avoided by keeping the rate of flow of air below a certain limit, and allowing about two minutes with the current off, between successive readings. To obtain definite humidities, the second water bottle was heated to some temperature below that of the corona tube, and it was assumed that the air was saturated at the temperature of the water. Since the rate of flow was always small, and the top of the bottle contained a consider-

able air space, this assumption seems to be justified. Before the air reached the corona tube, its temperature was raised, and consequently its relative humidity lowered, although the absolute humidity was unchanged. To insure this condition, the moist air was driven through the system for perhaps half an hour, or until the corona current had fallen to a steady value, before the first readings were taken. After each run with moist air, hot dry air was driven backwards through the corona tube and through the line as far as the water bottle, in order that when the temperature was subsequently lowered, there should be no condensation in the lines.

All observations were made at atmospheric pressure, which was about 74 cm of mercury.

Results

The results of the experiments are indicated in the following table, which shows the constants obtained for a number of current-voltage

TABLE I

Constant of equation: $i = \frac{C\,V(V-V_0)}{V_1-V}$

Group	No.	Temp	C (10^{-6} amp.)	V_0 (kv)	V_1 (kv)	k	moisture (per cent)
I	1	290°K	6.69	3.80	24.65	5.5	
	2	321	13.5	3.78	31.0	6.5	
	3	360	11.6	3.65	25.4	7.0	
II	4	298	4.98	2.94	22.9	3.3	
	5	417	8.99	2.55	29.6	4.4	
III	6	374	21.9	3.44	48.5	6.4	
	7	376	13.9	3.37	32.8	6.2	
	8	347	14.55	3.07	35.1	6.0	
IV	6	374	21.9	3.44	48.5	6.4	0
	9	376	7.51	2.63	30.6	3.5	44
V	7	376	13.9	3.37	32.8	6.2	0
	10	378	14.4	3.63	39.4	5.2	4.5
VI	8	347	14.55	3.07	35.1	6.0	0
	11	346	7.84	3.36	24.2	4.9	2.1
	12	346	8.20	3.42	27.4	4.0	8.3

The coefficient β, which has to do with the increase of the ionizing region, ranges from 0.00046 to 0.00105 cm. per e.s. unit of potential.

relations, some of which are represented on the curves. The mobilities were computed by means of the defining equations for C and V_1, which give, using electrostatic units,

$$k = \frac{CR^2}{2(V_1-V_0)} \log_e \frac{R}{r_0}.$$

For tabulation, the mobilities have been reduced to cm per sec. per volt per cm. Since the starting potential and the constants C and V_1 are very greatly affected by conditions of surface, only those runs should be compared that were taken within a short time of each other. Related runs are indicated in the table by the various groups, and comparisons should be made only *within* groups.

The effects of temperature and humidity are best indicated by the mobilities, and comparisons within groups are shown in Tables II and III.

TABLE II

Effect of temperature on mobility

Run numbers	Ratio of temperatures	Ratio of mobilities
1,2	1.19	1.18
1,3	1.24	1.28
2,3	1.12	1.085
4,5	1.40	1.34
6,8	1.08	1.06
7,8	1.08	1.04

TABLE III

Effect of humidity on mobility

Run number	Temperature	Per cent moisture	Ratio of mobilities*
9	376°K	44	0.55
10	378	4.5	0.84
11	346	2.1	0.82
12	346	8.3	0.67

*Ratio of mobility in moist air to that in dry air.

The accuracy with which the equation represents experimental data is indicated on the plots, where in each case the curve is drawn by the equation, and the circled points show the direct result of experiment. It is evident that the equation fits the observations well within the limits of experimental error, over a considerable range of temperatures and humidities of the air. A further test is furnished by the computed mobilities. It has been found[7] that over a wide range of temperatures the mobilities of ions in air are proportional to the absolute temperature; and Table II shows that within groups of related runs, the mobilities computed by means of this equation exhibit that proportionality. The values obtained for the mobilities, however, are from two to four times as great as those usually found by direct measurement, which are of the

[7] Phillips, P., Roy. Soc. Proc. **78,** 167 (1906)
Kovarik, A. F., Phys. Rev. **30,** 415 (1910)
Erikson, H. A., Phys. Rev. **6,** 345 (1915)

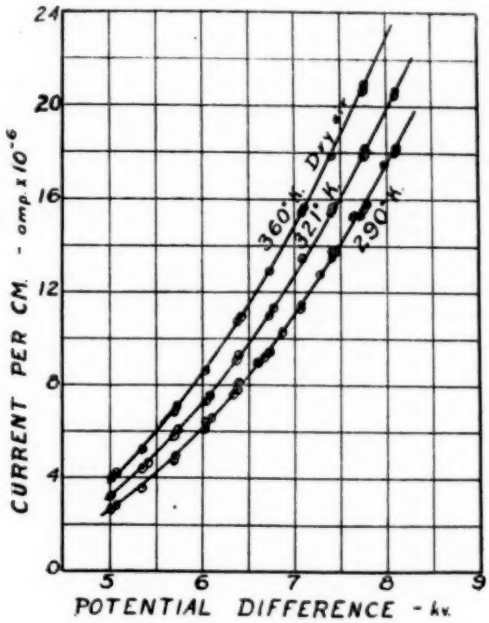

Fig. 1. Corona current curves for various temperatures, in dry air.

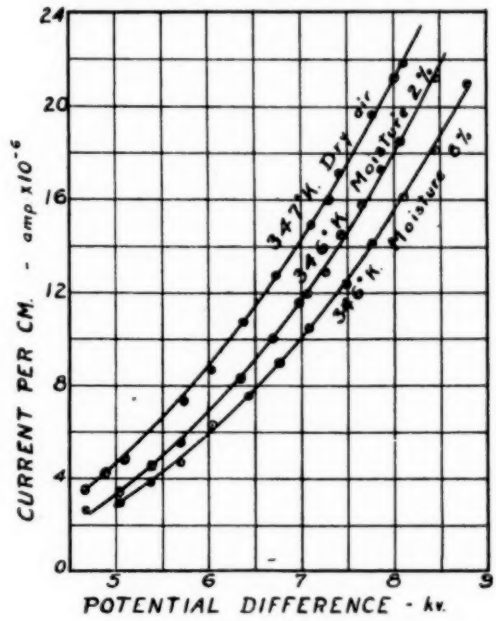

Fig. 2. Corona current curves for various humidities.

order of 1.6 to 2.0 (in terms of the volt) for the negative ion, and 1.3 to 1.6 for the positive ion.[8]

These high values of the mobility may be due to a combination of two causes. In the first place the effect of the space charge has not been completely taken into account. Its effect would be to diminish the field just outside the glow. The neglect of this effect would be to give values of the mobility too high as calculated from the equation. In the second place the presence of electrons would give rise to a higher value of k. Franck[9] has measured mobilities as high as 12.26 in cylindrical fields but his further investigation showed that this high value was due to the presence of electrons.

Table III shows the well known effect of water vapor in decreasing the mobilities.

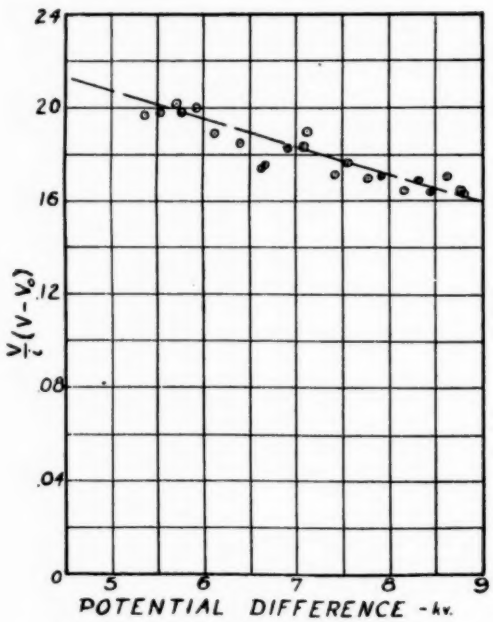

Fig. 3. $V(V-V_0)/i$ as a function of voltage.

Comparison with Previous Equations

Comparison of Eq. (6) with those of Almy, Schaffers, and Townsend, shows these points:

(1) Almy's equation fits experimental data fairly well over a limited range of voltages, but cannot fit over any wide range. According to his

[8] Kovarik, A. F., Roy. Soc. Proc., **86**, p. 154, (1912)
Lafay, A., Comptes Rendus, 173, p. 75, July, (1921)

[9] Franck, J. Ann. der Phys. **21**, 972 (1906)

equation, the quantity $V(V-V_0)/i$ should be a constant; while by Eq. (6) it is a linear function of the voltage. This relation is shown for a typical run in Fig. 3, in which, although the method of plotting brings experimental errors into prominence, it is clear that the value is not a constant, and a linear relation is at least a possible interpretation of the plot.

(2) Schaffers' equation fits experimental data satisfactorily over the range used, 5 to 9 kv, but yields mobilities as much as 100 times as great as those found by direct measurement.

(3) Townsend, by the use of his equation and Watson's[10] data, has computed mobilities in agreement with those found by direct measurement, but his equation is not very useful, because of its form.

(4) The equation developed in this article fits experimental data over the range of voltages used, and over a considerable range of temperatures and humidities. It possesses the following advantages over the other equations: it is accurate over a wider range of voltages than Almy's; is in a much more convenient form than Townsend's; and yields mobilities in rough agreement with known values, which Schaffers' equation fails to do.

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June 16, 1923.

[10] Watson, E. A., Electrician, **64**, 709, Feb. 11, (1910)

ROTATION IN A RAREFIED GAS FROM THE POINT OF VIEW OF THE KINETIC THEORY

By G. Breit

Abstract

Kinetic theory of the resistance to a sphere rotating in a rarified gas.—The analogous two-dimensional case is first treated. The method used is to replace the gas by a suitable model having a comparable mean free path, consisting of a number of concentric rough circles with molecules of negligible diameter traversing the intervening space and serving to transfer angular momentum from one circle to the next. The first order correction term in the expression for the m.f.p. is obtained, the moment on the model due to the rotation, and the corresponding coefficient of viscosity. The results are shown to apply to a two-dimensional gas and are readily extended to the analogous three-dimensional case, giving the moment of force L in terms of the ordinary hydrodynamic expression L_0, the m.f.p. λ, and the radius r: $L=(1-3\lambda/r)\,L_0$. The procedure used by Cunningham for the analogous translational case seems to be applicable here even though its validity for the translational case is questionable.

IN the modern theories of electrolytic conduction, the rate of polarisation of dielectrics and allied phenomena, a quantity is introduced which represents the damping influence of the medium on the ions or dipoles suspended in it. This quantity is then estimated by Stokes' Law or by the corresponding formula for rotation. It is of importance to know the accuracy of this estimate. For the case of translation the problem has been treated by Cunningham[1] and recently the rotational case has been discussed by Epstein.[2] An outline of a treatment of the rotational case will be given below, and a correction term to the hydrodynamical solution will be worked out. The treatment will then be compared with that of Cunningham and Epstein.

Our ultimate aim is to consider the rotation of an actual gas. The Kinetic Theory idealizes the gas by supposing that all the collisions are elastic. In order to facilitate the solution and to present it in its simplest form let us idealize still more by getting rid of the third dimension and confining ourselves to a two dimensional gas in the special form of a collection of elastic disks gliding over the surface of a plane. A perfectly rough circle is situated in the plane and is rotated about its center. We are asked to find the moment which must be applied to the circle in order to keep it rotating with a constant angular velocity.

[1] Cunningham, Proc. Roy. Soc. A **83**, 357, 1910

[2] Epstein, to appear in Phys. Rev. **23**, June 1924

The essential things which determine the moment are the size of the disk, the masses and velocities of the molecules, and their mean free path. For a rigorous solution it would be necessary to determine the distribution of the velocities and the relative numbers of molecules with different free paths. The formulas for the viscosity and the diffusion of gases seem to show, however, that the error committed by assigning the same free path to all molecules is not very great. Furthermore from a physical standpoint the free path computed from considerations relating to the effective cross section of molecules suffers from the indefiniteness of this cross section. For these reasons it may be pardonable to depart from the standard classical methods and to neglect the differences in the behavior of gases having different distributions of the free paths as long as the means are the same.

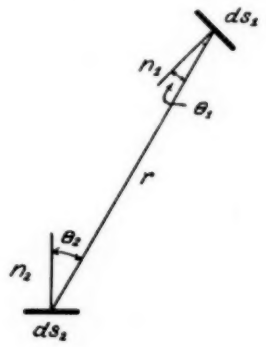

Fig. 1

This viewpoint enables one to substitute a system which is simpler for purposes of calculation than the elastic disks. The system adopted is one of concentric rough circles. The space between two circles is traversed by molecules of negligible diameter (a Knudsen gas) which transfer the angular momentum from one circle to the next. The collisions between the molecules are thus replaced by collisions with the circles.

(1). *Mean free path of a Knudsen gas between two rough concentric circles.* Each element of length is assumed to reflect during one second in the angle θ, $\theta+d\theta$ a number of molecules proportional to $\cos\theta\, d\theta$.

It may be shown that in an enclosure the walls of which possess this property, the same number of molecules fall on an element of length within a given angle as are reflected in the same angle and in the same time. To prove this let ds_1, ds_2, be two elements of the enclosure (Fig. 1) and let the joining line r make angles θ_1, θ_2 with the normals n_1, n_2. Let us suppose also that the laws of emission at ds_1, ds_2 are respectively

$\frac{1}{2}Ids_1 \cos\theta_1 d\theta_1$ and $\frac{1}{2}Ids_2\cos\theta_2 d\theta_2$. The number of particles sent into ds_2 by ds_1 is therefore $\frac{1}{2}Ids_1\cos\theta_1 d\theta_1 = \frac{1}{2}I\cos\theta_1\cos\theta_2 ds_1 ds_2/r$ since $d\theta_1 = ds_2\cos\theta_2/r$. The expression being symmetrical for the two elements we conclude that the same number is sent by ds_1 to ds_2 as by ds_2 to ds_1. This also implies that ds_1 sends as many particles into a finite angle as there are coming back to ds_1 through that angle; and further the number of molecules crossing an element of length in a given direction (not subtracting the number of molecules passing in the opposite direction) is the same as the number of molecules impinging on an equal element of length on the wall.

The mean free path is taken to be the mean of all the free paths laid off during a long interval of time. To calculate it, we find the mean free paths laid off per second by all the molecules starting at a particular

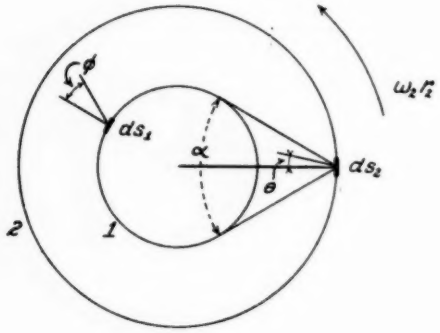

Fig. 2

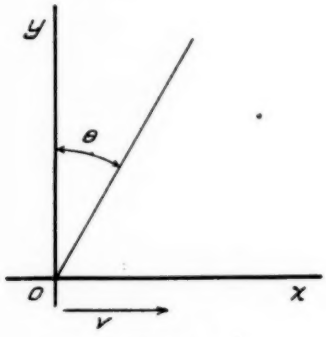

Fig. 3

element of length on the wall. We then take their sum and sum the results obtained for the various elements. We divide the result obtained by the total number of free paths laid off during a second.

Consider now the element ds_2 on the circle (2), Fig. 2. The particles leaving it strike either the circle (1) or (2) according as to whether their directions lie inside or outside the angle α. It is convenient in our final summation to have to deal only with the circle (2). For this reason we shall consider together with the free paths starting from ds_2 also certain free paths starting on the circle (1); and we shall be careful to consider these paths in such a way that all paths starting on (1) are accounted for once and only once by some element on (2). Now it is clear that any path starting on (1) must end on (2). Further all particles coming to ds_2 within the angle α come from (1) and no others can come to ds_2 from (1). Hence it is necessary to know only the free paths and the total number of particles coming to ds_2 within the angle α. We have

shown however that the total number of particles coming through α must be emitted through α. Thus in order to take into account the circle (1) while considering (2) it is sufficient to give the weight 2 to any particle reflected inside α and the weight 1 to any particle emitted outside α. By symmetry it will now suffice to obtain the mean free path for the element ds_2 without integrating over circle (2). Within the angle α the free path is $r_2\cos\theta-\sqrt{(r_1^2-r_2^2\sin^2\theta)}$.

Outside α the free path is $2\pi r_2\cos\theta$. Performing the necessary integrations it is found that the m.f.p. (l) is connected with the radial difference Δr by

$$\Delta r = r_2 - r_1 = 2l/\pi \tag{1}$$

(2). *Law of reflection from a moving rough surface.* It will be assumed that the surface of the rings is such that the velocity distribution referred to the moving surface does not depend on the velocity of the surface.

Let OX, OY be a set of fixed axes and let OX' OY' be axes moving with the surface. OX' is in the surface and OY' is perpendicular to it. Let the surface move along OX with the velocity v and let the velocity of the reflected molecule be c (with respect to OXY). The equations of the transformation are:

$$\begin{array}{lll} x'=x-vt; & x'=c'\sin\theta' t; & \tan\theta = x/y; \\ y'=y'; & y'=c'\cos\theta' t; & \tan\theta' = x'/y'. \end{array}$$

From these we find, neglecting the square of (v/c),

$$c = c'(1+\frac{v}{c'}\sin\theta)$$

$$\cos\theta'\, d\theta' = (1+\frac{2v}{c}\sin\theta)\cos\theta\, d\theta. \tag{2}$$

It may be shown that to the first order in v/c, the number of molecules leaving any element of length is equal to the number incident on it.[3]

(3). *Moment due to relative rotation of two rings.* We are interested only in very slow rotations. Thus we may suppose that the moment due to the relative rotation of two rings is a linear function of their angular velocities. Since the moment must vanish if both velocities vanish, the function is also homogeneous and is a sum of two terms. Therefore we may compute the moment on the inner ring when the outer rotates with angular velocity ω_2 and add to this the moment of the inner ring when the outer is at rest and the inner rotates with an angular velocity ω_1.

(a) Inner ring is at rest and the outer is rotating with an angular velocity ω_2 [see Fig. (2)]. The moment on circle (1) due to molecules

[3] I am indebted to Dr. J. H. Van Vleck for pointing out the necessity of a proof of this.

from an element ds_2 of circle (2) will be computed and then multiplied by $2\pi r_2/ds_2$ in order to get the total moment. The number of molecules having the velocity c and reflected from ds_2 in the direction $(\theta, \theta+d\theta)$, if ds_2 is at rest, is $\nu_c c/2\pi\cos\theta d\theta\, ds_2$ where ν_c denotes the number of molecules having the velocity c per unit volume. By Eq. (2), if ds_2 is in motion, the distribution of the molecules is given by $(\nu_c c/2\pi)[1+(2\omega_2 r_2/c)\sin\theta]\cos\theta\, d\theta\, ds_2$. The surface of the circle (1) being rough, the directions of reflection of any incident particle in the long run do not depend on the direction of the incidence. Thus on the average the momentum of the reflected particles is normal to (1) and therefore has no moment about the center. Consequently the moment about O exerted on (1) is simply the sum of the moments of momenta about O of the particles sent to (1) by ds_2 per second. For particles of the velocity c this is by (2) the integral from $\theta=-\sin^{-1}(r_1/r_2)$ to $(mc/2\pi)r_2\nu_c c ds_2\times\cos\theta(1+3\omega_2 r_2\sin\theta/c)\sin\theta\, d\theta$ which is equal to $(r_1^3/\pi r_2)mc\nu_c ds_2\omega_2$.

Letting $\bar{c}\Sigma\nu_c=\Sigma c\nu_c$, $\nu=\Sigma\nu_c$ and replacing ds_2 by $2\pi r_2$ the total moment is $2\nu m\bar{c}\omega_2 r_1^3$.

(b) Outer ring at rest and inner rotating with angular velocity ω_1. Consider all the particles falling on ds_1 in Fig. 2. Their directions before incidence are entirely random, therefore their resultant momentum is perpendicular to ds_1 and has no moment about O so that it suffices to compute the moment of the momenta of the reflected particles. A process quite similar to that discussed in (a) gives for this moment $-2\nu m\omega_1\bar{c}r_1^3$.

(c) Both rings rotate. Resultant moment is

$$L=2\nu m\bar{c}(\omega_2-\omega_1)r_1^3. \tag{3}$$

(4) *Law of motion of the model.* This is obtained by expressing the fact that the transferred moment is the same for all rings. The difference equation to be satisfied is

$$r_n^3\,(\omega_n-\omega_{n+1})=A \tag{4}$$

where A is independent of n.

It is clear that a constant may be added to ω without affecting the validity of Eq. (4). This means that a constant angular velocity may be given to all rings without requiring an external agency to maintain the motion. At present we shall be concerned with the particular solution for which the angular velocity is zero at infinity. This solution of (4) is

$$\omega_n=A\left(\frac{1}{r_n^3}+\frac{1}{r^3{}_{n+1}}+\ \cdots\right)=\frac{A\pi^3}{8l^3}\left(\frac{1}{n^3}+\frac{1}{(n+1)^3}+\frac{1}{(n+2)^3}+\ \cdots\right).$$

If n is large the series in parentheses is approximately equal to $1/2n_1^2 = \int_n^\infty dx/x^3$. As long as this approximation holds the angular velocity is proportional to the square of the distance from O and thus obeys the same law as that obtained in hydrodynamics. In order to obtain the next approximation we use the identity

$$\frac{1}{2n^2} = \left(\frac{1}{2n^2} - \frac{1}{2(n+1)^2}\right) + \left(\frac{1}{2(n+1)^2} - \frac{1}{2(n+2)^2}\right) + \cdots$$

and expand the terms into $(1/n^3)(1-3/2n+\cdots)$ so that the identity reduces itself to

$$\sum_n^\infty \frac{1}{n^3} \backsimeq \frac{1}{2n^2} + \tfrac{3}{2} \sum \frac{1}{n^4} \backsimeq \frac{1}{2n^2}\left(1+\frac{1}{n}\right).$$

Within this approximation the angular velocity is

$$\omega_n = \frac{B}{r_n^2}\left(1+\frac{\Delta r}{r_n}\right) \tag{5}$$

where B is independent of n.

For very large values of n the term $\Delta r/r_n$ becomes negligible and $\omega = B/r^2$. For such a case $\omega_n - \omega_{n+1} = -d\omega/dr)(2l/\pi)$ and by Eq. (3)

$$L = -(8lvm\bar{c}/\pi)\omega r^2 \tag{6}$$

This corresponds to the Stokes' formula. For small r the formula becomes incorrect but can be corrected by means of (5). In fact the moment transferred is the same for all rings. (The moment transferred is one of the two equal and opposite moments exerted on each ring from its two sides). At infinity (6) applies accurately. Thus it is only necessary to know $\lim (r\to\infty)\omega r^2$ which by (5) is $(1-\Delta r/r)\omega_n r_n^2$ so that

$$L = (1-\Delta r/r)L_0 \tag{7}$$

where L is obtained from (6). It is easily found that the coefficient of viscosity of our model is

$$\mu = 2lm\bar{c}v/\pi^2 \tag{7'}$$

so that (6) can be also written in the form

$$L = -4\pi\mu\omega r^2 \tag{7''}$$

this is the expression given in Lamb (Fourth edition, 1916, p. 580, Eq. 4).

So far we have been concerned only with the model. We are now about to discuss the application of the results to an actual gas. A modification of terminology will prove necessary for this.

In the above we denoted by l the m.f.p. in the space between two rough circles. This is not the m.f.p. of the gas which the model represents. The latter m.f.p. is $l/2$. This is inferred without difficulty from the discussion of the ambiguity of the m.f.p. in the derivation of viscosity

as given by Jeans Dynamical Theory of gases p. 289. It may be also seen as follows.

In the gas collisions take place between molecules. In the model collisions take place between molecules and rough walls. The rough walls thus act as partner molecules for the collisions. Since the walls may be stationary the number of partner molecules in the walls must be set equal to the number of molecules between the walls. The partner particles in the walls have practically a zero m.f.p. and zero velocity. The m.f.p. reckoned over the free and the partner molecules is then $l/2$ and $\Sigma\nu c = \nu' c'$ where $\nu' = 2\nu$, $c' = \bar{c}/2$ are respectively the actual number of molecules and their average velocity.

Thus if λ should denote the m.f.p. of the gas then Eqs. (7'), (1) give

$$\mu = 4\lambda m\bar{c}\nu/\pi^2; \quad \Delta r = (4/\pi)\,\lambda \tag{8}$$

and the correction factor is $(1-4\pi/\lambda)$ if the result is applied to the model.

But it would be wrong to regard this as an approximation to a body in a two dimensional gas. This case is fairly represented by a disk the radius of which is $(n+\frac{1}{2})\Delta r$ where n is an integer. For in this case the m.f.p. of the molecules striking the circumference of the disk is λ.

In order to obtain the correction factor, we express the condition of equilibrium of the ring of the system having a radius $(n+1)\Delta r$. Calling its angular velocity Ω and that of the disk ω the torque is by (7), (7''), (8) and (3)

$$L = (1-\Delta r/r)\ (8l\nu m\bar{c}/\pi)\ \Omega\ (r^2 + r\ \Delta r) = 2\nu m\bar{c}\ (\omega-\Omega)r^3 \tag{9}$$

which when solved for Ω gives

$$L = (1-2\Delta r/r)\ L_0 = (1-8\lambda/\pi r)\ L_0. \tag{9'}$$

(5) Accuracy to be expected of correction formula. If instead of dealing with the model one considers a two-dimensional gas in which the probability of a molecule passing a distance r without collision is $e^{-r/\lambda}$, the principle of transport in the calculation of viscosity may be applied and the value of viscosity calculated. The result of the calculation is

$$\mu = \tfrac{1}{2} m\lambda\bar{c}\nu. \tag{10}$$

This differs from (8) by the comparatively insignificant factor $\pi^2/8$. Thus the model is in this sense a fair representation of a two-dimensional gas.

(6) Generalisation to three dimensions. Let us consider Eq. (9) in more detail. The expression on the extreme right is the moment as calculated from the collisions of the molecules falling on the disk from the layer of gas in the immediate neighborhood of the disk. The central

expression is the moment calculated for an infinitely large ring on hydrodynamical considerations and corrected for the deviation from the hydrodynamical law of motion by the factor $(1-\Delta r/r)$. This correction factor just suffices to reduce $(r+\frac{1}{2}\Delta r)^2$ to r^2. Thus the moment of a layer of gas having a radius $(r+\frac{1}{2}\Delta r)$ and sending its molecules to the disk can be computed correctly by neglecting the two mutually compensating errors of the deviation from the hydrodynamical law of motion and of the inequality in the radii of the layer and the disk.

It is of interest to see more clearly the reasons for this fact. They are traced back to the evaluation of the $\Sigma_n^\infty n^{-3}$ which we found to be approximately $(1/2n^2)(1+n^{-1})$ or what is the same thing $\frac{1}{2}(n-\frac{1}{2})^{-2}$. That this should be the case could be inferred from a geometrical picture of the integral of $y=1/x^3$ if it is remembered that n^{-3} is the value of y in the middle of the interval $(n-\frac{1}{2}, n+\frac{1}{2})$.

The peculiar compensation of the two errors is therefore independent of the power 2 to which r is raised in the hydrodynamical expression.

If we should apply the method used to an analogous problem in three dimensions we should obtain a similar result as long as the elements (concentric spheres in this case) are equally spaced. It may be shown that they are actually equally spaced to the first order of λ.

These results show that the procedure used by Cunningham is likely to lead to correct results in our rotational case. The errors involved will be due to differences between an actual gas and the model.

Neglecting these errors we can solve the three-dimensional case quite easily. Eq. (9) may be written in the form

$$m\Omega\,\Delta r = r(\omega-\Omega) \tag{11}$$

where m is the number of dimensions. Thus for a sphere the correction factor is $(1-3\Delta r/n)$. By a simple integration $\Delta r=\lambda$ and the corrected moment is

$$L=(1-3\lambda/r)\,L_0. \tag{11'}$$

This formula is not directly comparable with Epstein's, because the constant λ is here connected with the viscosity μ by the relation $4\mu=Nm^c\lambda$ while in Epstein's case $\mu=0.3502Nm\bar{c}l$. Roughly this agrees with Epstein's (67) for $s=1$, though from the very nature of the two cases no precise comparison is possible. However (11′) and (11) correspond quite closely to Epstein's $C=a^3\omega/(1+3\mu/\beta a)$ and in this sense agree with his result. Our assumption as to the nature of reflection actually corresponds to $s=1$.

(7). *Bearing on the Stokes-Cunningham formula.* Cunningham[1] in his treatment of the analogous translational case uses a procedure the

correctness of which may be questioned. He considers the moving sphere as retarded by the action of the layer of gas immediately adjoining it which he considers as retarded by the surrounding gas, and he applies to the layer the uncorrected formula of Stokes setting the radius of the layer equal to the radius of the sphere.

It seems to the writer that neither the application of Stokes' formula to the motion of the layer nor the consideration of the equality of its radius with that of the sphere are correct to the necessary approximation (first order in l). However in the rotational case the two errors introduced appear to compensate each other and Cunningham's procedure seems to lead here to correct results to within the first order correction terms in the mean free path.

The writer wishes to acknowledge his great indebtedness to Prof. Ehrenfest of Leyden, under whose guidance he began the work on this problem, and to Prof. Lorentz of Leyden, who kindly went through some of the calculations.

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ELECTROLYTIC CAPACITY AND RESISTANCE OF PT-RHODAMINE B-PT CELL, AS DETERMINED BY A PHASE ANGLE METHOD OF MEASUREMENT

By Kuo-Feng Sun

Abstract

Direct measurement of the phase angle of any reactive circuit.—The circuit is placed between A and B in one arm of a bridge, an adjustable resistance R between B and C, two equal resistances between A and D and between D and C. An e.m.f. E is applied at A and C, and the potential difference between B and D is determined by applying a known e.m.f. in quadrature with E, and adjusting for zero current. For highest sensitiveness, R should equal the impedance of the reactive circuit. The sensitiveness increases with decreasing frequency. The method is *useful in testing any auto-transformer.*

Sensitive null instrument for alternating currents.—A direct current galvanometer was connected in series with a synchronous commutator, permitting independent adjustment of E and R for zero current.

Electrolytic capacity and resistance of Pt-Rhodamine B-Pt cell—was measured by the phase angle method for various cells with electrodes of solid *Pt* or of cathodically deposited films of various thicknesses in various relative positions. The results can be regarded as correct only when the electrodes have relatively small resistance. For the case of two parallel plates, the correction for the resistances of the electrodes is made. *Variations with e.m.f. and frequency.* The resistance is constant but capacity increases with e.m.f. for values above 0.4 volts and decreases with frequency in proportion to $1/\sqrt{f}$ for 10 to 70 cycles. *Variation with arrangement of electrodes.* A cell with parallel solid electrodes has a greater capacity than one with the same electrodes lying in a plane. The capacity increased as the thickness of the sputtered film decreased and was greater for films with a black color. *Effect of increasing concentration and temperature* was to increase the capacity and decrease the resistance.

Introduction

THAT an electrolytic cell acts as a network of capacity and resistance has long been known, Varley[1] and Kohlrauch[2] having apparently been the first to undertake their determination. Since then the accurate measurement of this capacity and resistance has been the subject of many investigations.[3] The precise measurement of the electrolytic capacity of high resistance cells has, however, been undertaken by only a few. Pro-

[1] Varley, U. S. Patent Specification, Electric Telegraph, etc., Jan. 1860
Phil. Trans. **161**, 129, 1872

[2] Kohlrauch, Nachr. der K. Gött. Gesell., p. 453, Sept. 1872;
Pogg. Ann., **148**, 143, 1873

[3] Colley, Wied. Ann. **7**, 206, 1879;
Hopkinson, Wilson and Lydall, Proc. Roy. Soc. **54**, 407;
Sheldon, Leitch and Shaw, Phy. Rev. **2**, 401, 1895; and others.

fessor C. C. Murdock[4] in his recent work on the photo-active electrolytic cell, Pt-Rhodamine B-Pt, described a direct current method for measuring the capacity of such a cell. This method does not, however, yield results of high accuracy.

Since the large time constant, or small phase angle, of a high resistance cell makes impracticable most of the methods usually employed, it was suggested by Professor F. Bedell that it would be desirable to obtain a sensitive and accurate method for the direct measurement of this small phase angle. The author, accordingly, undertook the development of such a method for the accurate determination of the equivalent electrolytic capacity and resistance of high resistance cells. By this method it was hoped to be able to determine the electrolytic capacity and resistance of Rhodamine B cells, such as Professor Murdock was using, and to note the variation of these quantities with the impressed electromotive force, the geometrical construction of the cell, the con-

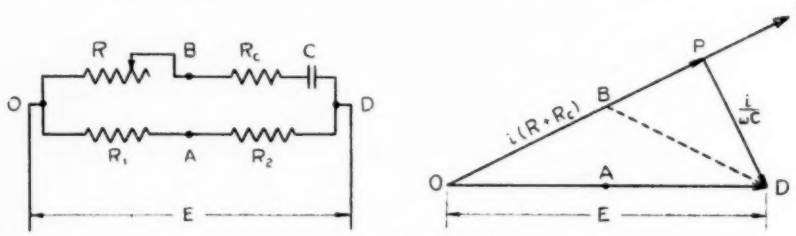

Fig. 1

centration of the solution, the temperature, and the thickness of the semi-transparent platinum film electrodes. In the process of this work it became desirable to separate the resistance of the electrodes from the resistance of the electrolyte. A method was developed for computing the true resistance and the capacity of the cell from the value of equivalent resistance and capacity as measured.

Part I. Method

Let an alternating electromotive force E be impressed upon two parallel circuits, as shown in Fig. 1. Let one of these circuits consist of a variable non-inductive resistance R in series with another non-inductive resistance R_c and a capacity C; let the other consist of two equal similar resistances R_1 and R_2 in series. The potential drop in the first circuit OBD, due to the resistances R and R_c, may be represented by a vector OP having a value $(R+R_c)i$, where i is the current flowing in the circuit OBD. The drop, $i/\omega C$ due to the capacity C is in quadrature with OP and is repre-

[4] Murdock, Phy. Rev. **17**, 626, May, 1921.

sented by the vector PD. Here ω is 2π times the frequency. The total drop is, accordingly, represented by the vector OD, which is equal to the impressed electromotive force E. As the current flowing in the resistance circuit OAD is in phase with the impressed electromotive force E, the potential drop through the resistances R_1 and R_2 must coincide with and be equal to the electromotive force E. Since $R_1=R_2$, the point A must be at the middle point of OD, as is shown in Fig. 2.

When R is varied while E, R_c and C remain constant, the electromotive force diagram may be represented[5] as in Fig. 2. The phase angle θ in the impedance circuit BD is constant as long as the values of R_c, C

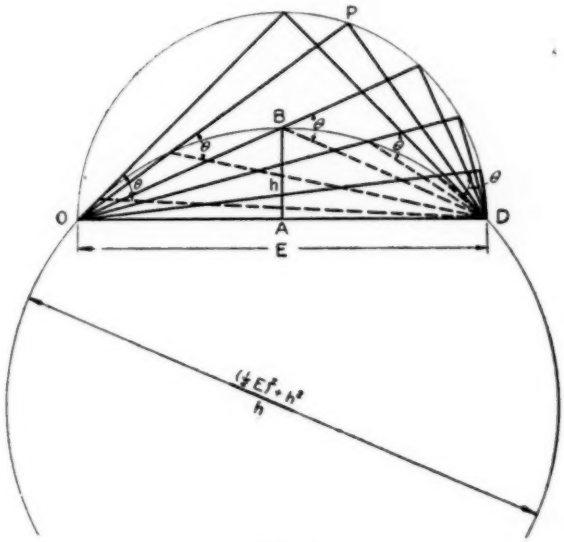

Fig. 2

and ω are constant. Hence, θ is not affected by the variation of R, and the angles supplementary to θ are all equal. The locus of the point B will, accordingly, lie on an arc of the circle passing though O and D. The diameter of this circle is $(\frac{1}{4}E^2+h^2)/h$ where h is the length of the perpendicular drawn from the middle point A of the line OD to the middle point B of the arc OBD. By the geometry of the figure, the perpendicular h is the shortest distance from the point A to the arc OBD.

If there were no capacity in the circuit BD (see Fig. 1), a point B could be found so that no current would flow through a detector connected between this point and the middle point A of the other parallel circuit. With capacity, however the electromotive force at the terminals of the

[5] Bedell and Crehore, Alternating Currents, p. 275;
LaCour, Theory and Calculation of Electric Currents, p. 49; and other texts.

detector can only be reduced to a minimum value h in quadrature with the electromotive force E, the magnitude of h depending upon the value of E and the phase angle θ. If, then, we had an idiostatic voltmeter that would read to very small fractions of a volt with high accuracy, we should have only to adjust for a minimum and read the value of h. E and R being known, the values of θ, R_e and C could then be obtained. As such an instrument does not seem to be available, the use of a balancing or zero method with a suitable detector seems most desirable.

In precise direct current measurements, zero methods have been used almost to the exclusion of others. The necessity for such methods is, however, even greater in alternating current measurements: first, on account of the limitations of alternating current indicating instruments; and second, on account of the practical impossibility of obtaining alternating currents as steady as the direct currents furnished by the storage battery. An electrodynamometer lacks sensitiveness as the current approaches zero. By separately exciting one of its coils, the sensitiveness may be greatly increased but means must then be provided for bringing the excitation in phase with the current to be measured. Used as a zero detector, a telephone is very sensitive under proper conditions, and has the advantage of simplicity and cheapness; but the capacity effects of the various parts of the circuit reduce the accuracy at low frequencies on account of the harmonics present. The vibration galvanometer is also very sensitive as a zero detector, when properly tuned to the fundamental or other harmonic to be balanced; but it is not satisfactory as regards ease of manipulation, and with a change of frequency there is a great decrease in sensibility.

Direct current galvanometer, with synchronous commutator, as detector. A direct current galvanometer, in combination with a reversing commutator[6] driven by the generator or by a synchronous motor, makes possible the same accuracy in alternating as in direct current measurement. In the present investigation, a 1000-ohm galvanometer, with a sensitiveness of 1.7×10^{-6} volt per millimeter of the scale at a distance of one meter, was used. The rectifying commutator used was fully described in the paper by F. Bedell.[6] It has four segments, alternately connected, and is directly driven by a four-pole synchronous motor. The alternating current to be measured is connected to the commutator segments through slip rings. Direct current is taken by two brushes bearing upon the commutator and accurately set so as to be separated from each other

[6] Devised by Ayrton and Perry, Phil. Mag., **12**, 297, 1906; see also: L. T. Robinson, Trans. A. I. E. E. **28**, 1024, June, 1909; C. H. Sharp and W. W. Crawford, A. I. E. E. **29**, 1517, July, 1910; F. Bedell, Jour. Franklin Inst., p. 385, Oct. 1913.

by a distance of one commutator segment. These brushes are so mounted that, without changing the distance between them, they can be shifted by a tangent screw and their position read upon a graduated scale. By such a device the brushes can be adjusted to any position with respect to the poles of the motor so that the reversal occurs at any desired phase of the current. By shifting the brushes to a proper position, the galvanometer may be made to respond to any given component of the current while it is insensitive to the component in quadrature with it. It will be noted that each degree of mechanical shifting of the brushes corresponds to two degrees of electrical phase shifting, when, as in this case, the driving motor has four poles.

It was suggested by Professor F. Bedell[6] that the resistance of brushes and contacts should be as low and uniform as possible. The fact was pointed out by C. H. Sharp and W. W. Crawford[6] that the apparent resistance of the sliding brush contact under working conditions tends to become very high as the current falls to a low value. To reduce this resistance, the brushes were made of seven strips of soft copper soldered at the end remote from the surface, which is a bevelled edge.

The commutator was connected to the motor by a fibre shaft, 20 cm long, which gave high insulation. The direct current for the field of the motor was brought by a circuit separated as far as possible from the testing circuits. The bearings were carefully fitted to eliminate axial motion and a small fly-wheel was used to eliminate hunting.

Details of method. The method of introducing a variable alternating current electromotive force between the points A and B in quadrature with the impressed electromotive force E, and balancing the voltage h by means of a zero detector connected between A and B was tried with various arrangements of the apparatus.

The arrangement finally used is shown in Fig. 3, and more completely in Fig. 4. As in Fig. 1, the variable resistance R is in series with resistance R_c and capacity C; in parallel therewith are two equal resistances R_1, R_2 in series. The electromotive force E applied to these two circuits in parallel, is brought through a reversing switch S_R (see Fig. 4) from adjustable taps on an auto-transformer 3′, 4′, which is supplied with current from taps 3, 4, on an another auto-transformer 1, 2, 3, 4, 5, having four equal coils. Taps 2, 4, are connected to the supply, in this case the A. C. end of an inverted rotary converter, the speed of which, and hence the frequency of the alternating current as shown by a frequency indicator, is controlled by field adjustment.

In order to measure h, a known alternating electromotive force was introduced between A and B which was adjusted until a zero reading was

obtained on the zero detector, consisting in this case of the direct current galvanometer and commutator already described. Various arrangements were tried, whereby the known electromotive thus introduced should be in quadrature with the terminal electromotive force E. This quadrature electromotive force was most satisfactorily obtained from the secondary of a Brookes inductometer M, connected as shown, having a range of 1 to 10 milli-henries, the primary and secondary each having a resistance of 4.5 ohms. In taking readings, the brushes of the synchronous commutator are properly set and M is adjusted until the galvanometer reads zero.

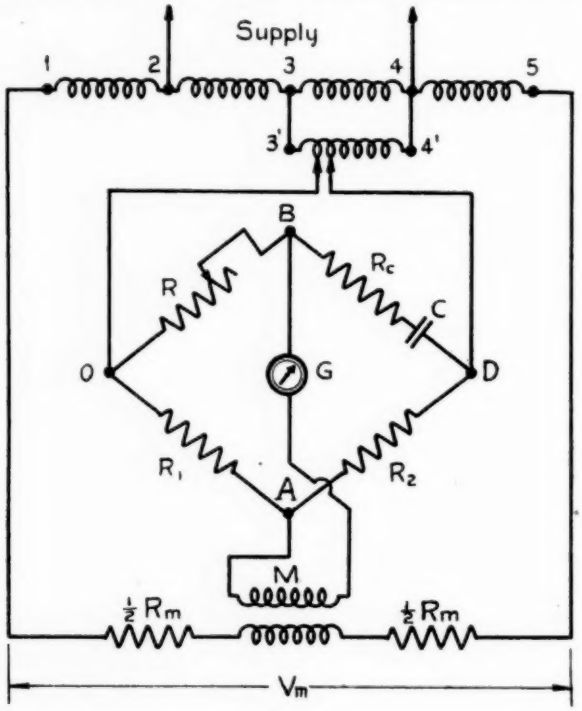

Fig. 3

The secondary inductometer voltage (and hence h) when the galvanometer reads zero, is $M\omega I_1$. The value of M is read from the inductometer; the value of I_1 is calculated from primary resistance and voltmeter reading.

A high non-inductance resistance was placed in series with the primary so that the primary current would be in phase with E. Hence, the secondary electromotive force, in quadrature with the primary current was in quadrature with E, as desired. It was found that accurate readings were obtained only when the primary resistance was divided into two equal parts symmetrically placed as shown.

Elimination of error. The complete connections are shown in Fig. 4. With the galvanometer connected to A (instead of B) and inductometer to B (instead of A) a slight difference in readings was found. This was made small by reducing as much as possible leakage and induction between different parts of the system. Errors due to thermal and contact electromotive forces in the commutator were eliminated by averaging the readings taken in the two positions. A Wagner[7] earth connection, through equal resistances r_1, r_2, as shown, was introduced to eliminate

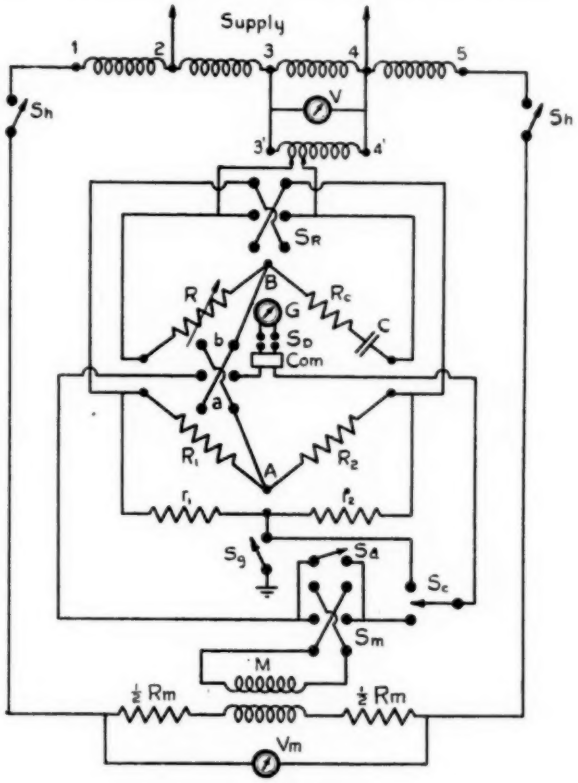

Fig. 4

a small zero deflection of the galvanometer. It is important that R_1 and R_2 be precisely alike. Curtis resistances were used for this purpose and were interchanged by the switch S_R.

Operation. As a preliminary, r_1 and r_2 were balanced against R_1 and R_2 by adjusting r_1 and r_2 until the galvanometer read zero. The commutator brushes were set so that the galvanometer was most sensitive for change of resistance. Switches S_G, S_A, S_M are open; ab is closed at a;

[7] K. W. Wagner, Elect. Tech. Zeit. **32**, 1001, 1911.

S_c is up. The main measurement is then taken. Switches S_G, S_M are closed; S_c is closed. A resistance balance, R, R_c against R_1, R_2 is first obtained by adjusting R. The commutator brushes are in the position of maximum sensitiveness for resistance adjustment; no change of M then affects the galvanometer. The brushes are then shifted ninety electrical degrees, giving maximum sensitiveness for M and zero sensitiveness for R. The value of M for zero galvanometer deflection was thus determined. By using the various reversing switches, eight readings for M and four for R were taken and averaged. Two or three minutes were sufficient for a set of readings. By this method, made possible by the synchronous commutator, the adjustments of M and R are separately made and are entirely independent of each other. With most zero detectors there is not this independence, and a tedious adjustment of M and R simultaneously is necessary in order to bring the galvanometer to zero.

A careful study of the conditions for sensitiveness shows that for maximum sensitiveness $OB=BD$, the resistance in series with the cell being equal to its impedance. The sensitiveness increases with decrease in frequency and increase in electromotive force so long as θ is less than 45°. The method, therefore, is well adapted for use at commercial frequencies, whereas a higher frequency is commonly required by other methods.

The accuracy of the method was tested under the conditions which apply when the capacity of cells is being measured. This was done by putting a known capacity of 1 microfarad in series with various known resistances and then measuring the values. The error in the measured value of C was found to be less than 1 per cent for values of R_c up to and including 20,000 ohms. For $R_c=40{,}000$ ohms, a 3 per cent error in the value of C was observed. The error in the measured value of R_c was found to be less than 1 per cent for values of R_c from 1,000 up to 40,000 ohms.

Part II. Measurements of the Capacity and Resistance of the Electrolytic Cell, Pt-Rhodamine B-Pt

Description of cells used. Most of the experiments on the photo-active properties of the electrolytic cell Pt-Rhodamine B-Pt, have been performed with cells in which the two electrodes were rectangular strips of platinum mounted side by side in the same plane.[8,9] Some work has also been done with electrodes mounted parallel to each other.[4] The measure-

[8] Goldmann, Ann. der Phys. **27**, 449, 1908

[9] Jenkins, Phys. Rev. **18**, 402, 1921.

ments here described were made using solutions of Rhodamine *B* in absolute ethel alcohol as electrolytes. Four types of cell have been used.

Type A. These cells had parallel rectangular electrodes of approximate area 1 cm^2 mounted .85 cm apart so as to form the opposite sides of a glass cell of approximate dimensions $0.5 \times 2.0 \times 0.85$ cm. There were small holes at the top and bottom so that the cell was filled when dipped into a solution. There were five cells of this type; cell A having strips of bulk platinum for electrodes and cells A_1, A_2, A_3, A_4 having semi-transparent sputtered platinum films of various thickness.

Type B. These cells were the same as that used by Mrs. Jenkins[9] except for the electrodes which were new. Two cells of approximate area 1 cm^2 for each electrode were used, B_2 having somewhat thicker electrodes than B_1.

Type C. The electrodes of cell *C* consisted of two parallel platinum strips of 1.662 cm^2 each. These were sealed into the ends of two glass tubes of small bore, and contact made by mercury columns in the tubes. During experiments these two tubes were so clamped that the electrodes were parallel and opposite. The distance between them was 1.652 cm.

Type D. The cells of type D consisted of two similar platinum electrodes with very narrow necks side by side on a piece of microscopic glass. The necks were made thick enough to make a good connection between the electrodes and the wires soldered at their ends. The distance between their nearest edges was 4 mm. During measurements, the cell was dipped carefully into the solution, so that the upper level of the solution just touched the bottoms of the necks of the platinum strips. This prevented any error contributed by the necks. There are seven cells of this type, cell D having bulk platinum electrodes and cells D_1 to D_6 having transparent electrodes of various thickness.

Decomposition voltage. In order to obtain some idea of the size of the impressed electromotive forces which might be safely used in the capacity measurements, preliminary experiments were performed on the decomposition voltage. The cell was polarized for one minute by impressing on it various potential differences and the corresponding currents were observed. At temperature 0°C the decomposition voltage of a 5.6 per cent solution was found to be about 1.0 volts. It increased upon dilution to about 1.3 volts for a .57 per cent solution. In these experiments the residual current was always large and in no case was the knee in the curve sharp. This was more particularly marked at higher temperatures.

Capacity as function of impressed voltage. A series of experiments was performed to find the variation of the electrolytic capacity with impressed potential difference. Fig. 5 shows the results obtained with cell B_1 filled

with a 6.26 per cent solution. The measurements were made at 0°C. The results shown are typical of those obtained with other cells and concentrations. The capacity was found to be practically independent of the impressed potential for values below 0.4 volts. For values of the order of 0.1 volts, however, large experimental errors were in evidence. These appear both in the capacity curve and in the resistance curve of Fig. 5. It was apparent from these results that an impressed potential difference of the order of 0.3 volts should be used. The electrolytic resistance was found approximately constant under varying electromotive forces.

Theory of equivalent resistance and capacity. When semi-transparent metallic films are used as electrodes, the resistance along an electrode may become comparable with the impedance of the cell and the distribution of

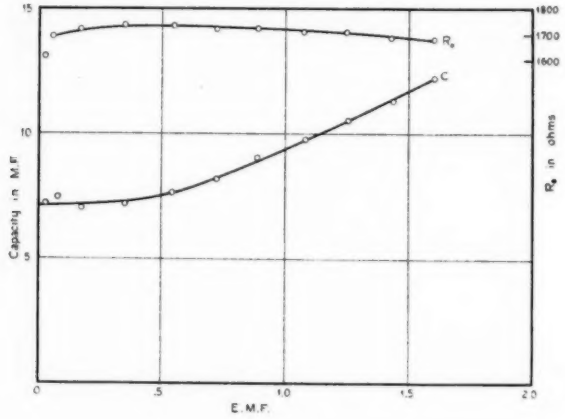

Fig. 5

the current leaving the surface of the electrode may be far from uniform. Under these conditions it would seem doubtful if the measured values of the resistance and capacity of the cell are the characteristic values desired. In cells of types B and D this variation occurs not only along the length but also along the width of the electrode. The computation of the correction for these cells would be complicated. This was the reason for the construction of the cells of the type A for which it has been found feasible to compute the correction. The general relation between the measured resistance R and capacity C and the corrected values R_0 and C_0 was found to be

$$R_0 - j\frac{2}{\omega C_0} = \left(R - j\frac{2}{\omega C}\right)\frac{1}{m}\tanh m$$

where $m = \sqrt{r/[R_0 - j(2/\omega C_0)]}$ and r is the sum of the resistances of the two electrodes. When $m < 1$, practical correction equations can be obtained

by expanding tanh m as a power series and neglecting the high powers. This gives, if powers higher than the second are ignored,

$$R_0 = R/(1+n) - \tfrac{1}{3}r\ (1-n) \quad \text{and} \quad C_0 = C(1-n)$$

where $$n = \frac{(\frac{1}{3}r)^2}{(R_0 - \frac{1}{3}r)^2 + (2/\omega C_0)^2} = \frac{(\frac{1}{3}r)^2}{(R - \frac{2}{3}r)^2 + (2/\omega C)^2} \text{ (approx.)}$$

In the work here described this degree of approximation was found to be sufficiently accurate.

Variation of capacity with type of cell. A large number of measurements of the electrolytic capacity and resistance of the cells described were made. Freshly made solutions of different concentrations were used in these experiments. A beaker containing the cell to be measured and a thermometer was placed in an ice bath. When the thermometer indicated 0°C, the experiment was started. The concentration of each solution was carefully determined just after its measurement.

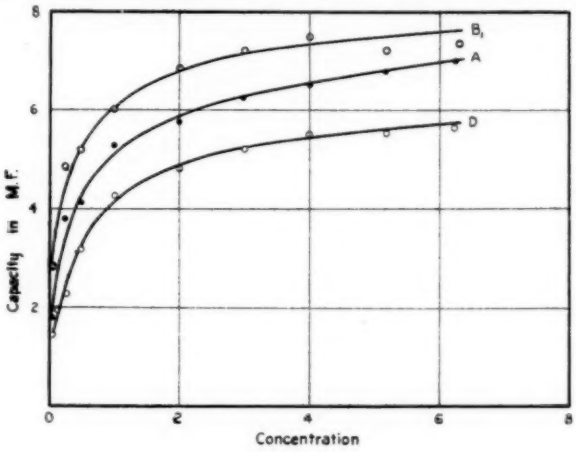

Fig. 6

It was found that all the cells gave the same general type of curve for the relation between the capacity per cm² of electrode and concentration. Fig. 6 shows typical results obtained with cells A, B, and D. The capacity increases rapidly at low concentration, but, after a certain concentration has been reached, it increases slowly. The decrease in the capacity with dilution of the solution is agreed upon by most investigators. The potential impressed upon the cells in these experiments was .356 volt.

In curves of Fig. 6, we see that the capacity per cm² of cell A is always greater than that of cell D even though they were measured under exactly the same conditions and both electrodes were cut from the same piece of platinum. The difference is evidently due to the position of the electrodes. As the electrodes of D lie in a plane, the current density

distribution may be far from uniform along the width of electrode and the relative effective area might be reduced. We also see that cell B_1 having sputtered film electrodes has higher capacity than that of D having bulk electrodes even though the conditions under which they were measured and their electrode positions were the same. This inequality in capacity may be explained by saying that, the effective area of the film electrodes is increased by the non-uniformly sputtered particles.

Capacity as a function of concentration. Because of the construction of cell A we can from its dimensions and from the value of the electrolytic resistance R_0 compute the conductivity of the solution. Fig. 7 shows the electrolytic capacity per cm² of electrode and the conductivity divided by

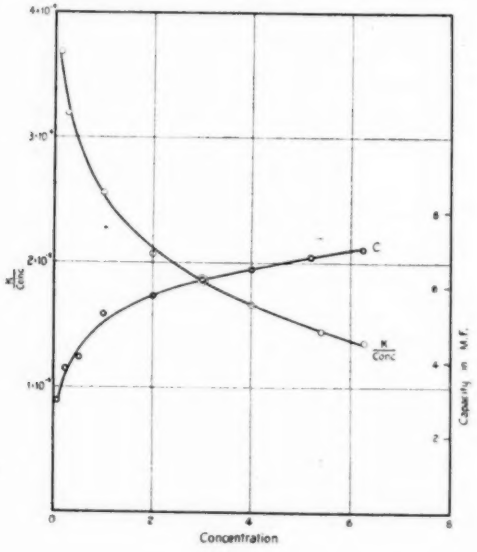

Fig. 7

the concentration as functions of the concentration. The latter is very nearly proportional to the equivalent conductivity. These measurements were also made at 0°C and with an impressed potential difference .355 volt.

Variation of resistance and capacity with temperature. A series of experiments was performed to investigate the effect of temperature upon the electrolytic resistance and capacity. A metallic vessel inside a larger vessel with non-conducting material between them was used as a water bath, in which was placed water of any desired temperature. Inside the bath a propeller type stirrer driven by a small motor was provided in order to make the temperature of the water uniform. The beaker containing the cell to be measured and a thermometer was dipped in the water and another thermometer was provided just outside the beaker.

As soon as these two thermometers indicated the same temperature the experiment was started.

The cells A, A_1, A_3, A_4, B_1, B_2, and C were measured. They have the same characteristics; the electrolytic capacity per square centimeter of electrode increases with the temperature and the electrolytic resistance R_0 (in case of cell B_1 and B_2, the equivalent resistance R') decreases with the temperature. The negative temperature coefficient of resistance is of the same order of magnitude usually found for aqueous solutions of inorganic salts.

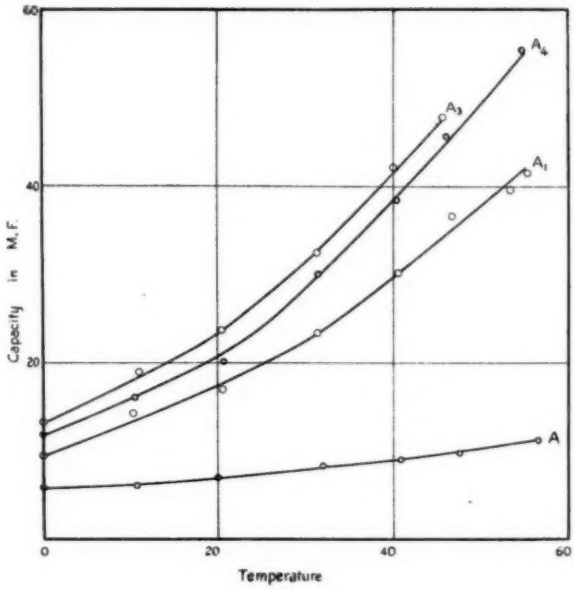

Fig. 8

Capacity as a function of temperature and electrode density with a note on the color of A_3. The relation between capacity per cm^2 of electrode and temperature for cells of type A is graphically represented in Fig. 8. It is to be noted that the temperature coefficient of capacity is positive. Similar results were obtained with type B and C cells.

Fig. 8 shows the comparison of the capacities of cells A, A_1, A_3, and A_4 when measured under the same conditions.

An indication of the relative thickness of the film electrodes was obtained by measuring the percentage of light transmitted by each. A Weber photometer was used for the purpose of this measurement. The transmission of the films were: A_1, 44.4 per cent, A_3, 14.6 per cent; A_4, 13.9 per cent. Curves A_1 and A_4 of Fig. 8 seem to indicate that the electrolytic capacity increases with the thickness of the film electrode. Experiments with five cells of type D also clearly indicate this conclusion.

Curve A_3 would seem to be an exception to this rule. The same is true of the data taken with cell D_5. The capacity of both these cells is larger than would be expected from the density of the electrode. The color of the electrodes in cells A_3 and D_5 is markedly darker than in the other cells. It will be seen also that the capacity of cell A with bulk platinum electrodes is less than that of the cells A_1, A_2, or A_4 with sputtered film electrodes.

An explanation of these apparent exceptions may be found in the fact that in a sputtered film we have the metal deposited in small particles. It may then be supposed to have taken on to some extent the properties of platinum black which has a very large electrolytic capacity. The darker films may have been deposited under conditions which augment this effect. In all the sputtered films, however, it seems to operate to such an extent as to make the electrolytic capacity greater than that of bulk platinum. By comparison of the resistivity of A_3 with others of Type A when they were measured under same conditions, it was found that the dark color of A_3 has no effect on the electrolytic resistance.

Variation of capacity with frequency. Measurements were made using cells of type D of the electrolytic capacity at frequencies varying from 10 to 70 per second. Within this range the capacity was found to vary inversely as the square root of the frequency. This agrees with recent results obtained by Jolliffe[10] with sulphuric acid at radio frequencies. Because of dependence of the capacity on frequency, the comparison of the results here obtained with those obtained by Murdock[4] by his direct current method is not particularly significant. The agreement, however, is surprisingly good. He obtained a value of 26 microfarads per cm^2 at room temperature with a 6 per cent solution and electrodes which transmitted 70 per cent of the incident light. The results here obtained indicate a lower value than this. In Fig. 8 for instance we find a value of 17 microfarads per cm^2 for a 5. 7 per cent solution with electrodes that transmit 44 per cent of the incident light.

In conclusion, I wish to express my hearty appreciation of the intensive interest and tireless encouragement given me by Professors Bedell, Murdock and Merritt and for the cooperation of Professors Gibbs and Richtmyer and of Mr. Holmes in obtaining all the necessary material to work out this problem. The kindly advice given by Professor Karapetoff of the Electrical Engineering Department is likewise greatly appreciated.

PHYSICAL LABORATORY,
CORNELL UNIVERSITY,
October 9, 1923.

[10] Jolliffe, Phys. Rev. **22**, 293, 1923

THE DIELECTRIC PROPERTIES OF WATER FOR CONTINUOUS WAVES

By G. C. Southworth

Abstract

Direct measurement of the refractive index of water for continuous waves of length about 2 meters.—In the method adopted continuous standing waves on wires were measured both in air and in water, and the refractive index determined as the ratio. Errors involved in the measurement of the length of waves in air due to imperfect reflection and to the resistance of the guiding wires, were investigated. In the case of water it was found that measurements varied with the size of the tank unless the waves were guided into the water by conductors extending through the tank. The final arrangement is analogous to Kundt's tube for sound waves; the level of the water in a vertical tube through which the parallel wires passed, was varied and the nodes determined from the variation of the current from a thermocouple placed in the bridge of the Lecher system, located at the lower end of the water column. Thus a number of nodes were located. The mean result for the refractive index comes out $8.88 \pm .04$, in good agreement with values obtained by others using discontinuous waves. This indicates that within the range of frequencies investigated, the group velocity of waves through water is the same as their phase velocity, within the accuracy of the measurements. No appreciable dispersion for the range of wave-lengths 124 and 276 cm was observed.

An oscillating circuit for producing short continuous Hertzian waves of length varying from 1 meter up, is described, in which the tube capacity is used as a means of coupling.

Introduction

AFTER Hertz's experimental verification of the existence of electric waves, much experimental work was done on the properties of different substances for propagating these waves. It was found that the index of refraction of liquids could be investigated very readily and in the course of these investigations it developed that water possessed an unusually high refractive index. The methods used consisted either of comparing the velocities of waves through space and through the liquid being considered or else the measurement of the capacity of a condenser before and after the liquid had been used as a dielectric. The first of these methods generally led to a value of about 9 for the refractive index of water although lower values were frequently obtained.

In all of these cases the source of waves was damped and free oscillations were used for the measurements. It may be shown that the velocity of propagation of groups of damped waves through an absorbing medium may be quite different from that for continuous waves of a simple sinu-

soidal form. This is more readily understood when it is considered that a series of groups of waves may be expressed as a Fourier series consisting of component frequencies. If the velocities of some of these components are different from the velocities of others, the resultant velocity will not represent that of the most prominent frequency as has usually been assumed. This phenomenon should be most pronounced near an absorption band.

Information concerning the dielectric properties of water for continuous waves is very limited. The only work done heretofore in this range was at a single frequency and apparently little effort was made to eliminate errors.[1] Also the available data on the index of refraction for

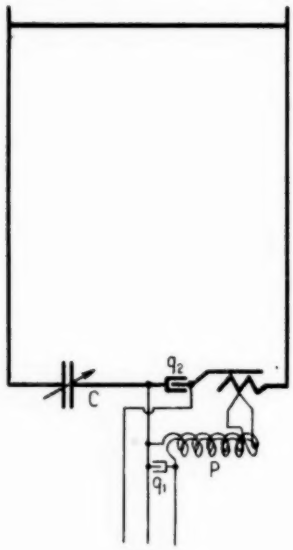

Fig. 1

damped waves might not be reliable for reasons mentioned above, therefore it was decided to repeat the classical experiments altering the methods to make them adaptable to continuous waves.

The electron tube oscillator has provided an ideal source, not only for radio frequency measurements, but also for ultra-radio frequencies. In order to make it adaptable to these high frequencies, a special circuit is necessary. A complete description of such a short wave generator has been published in the Radio Review.[2]

The circuit diagram of this oscillator is shown in Fig. 1. The oscillatory portion of the circuit is indicated by the heavy lines. Its frequency is determined by the dimen-

[1] M. Sauzin, Comptes Rendus, **171,** July 19, 1920
[2] G. C. Southworth, Radio Review, **1,** September, 1920

sions of the rectangle, the capacity C and the capacity between the electrodes of the tube. The relative location of the apparatus in the circuit is of very great importance. The conductors through which the input is led, place three points of the circuit at earth potential. It is preferable therefore to locate these three points as close together as possible. Since the length of the rectangle is appreciable compared with the wave-length, waves are propagated along the rectangle and reflected at the ends. The ground imposed by the power leads may well be placed near the middle of the end of the rectangle. If the capacity of C is approximately equal to that of the tube, the voltage distribution along the two sides of the rectangle will be symmetrical. The wave front is therefore approximately perpendicular to the conductors forming the sides of the rectangle. This symmetrical condition may be verified by grounding corresponding points on the opposite sides of the rectangle and observing the power output. A second point at earth potential will be observed near the middle of the closed end. Other circuits used for developing high frequencies are described in the articles mentioned below.[3]

In addition to the tubes known in the Signal Corp as VT-12, and VT-14, the G. E. type UV-202 may also be used very successfully. It is very desirable to remove the base of the tube in order to minimize the capacity between the electrodes. This device will give any wave-length from 1 meter up to those produced by electron tube circuits consisting of localized inductance and capacity. The advantages of this type of circuit can hardly be over-estimated. It has been shown that its constancy for both frequency and amplitude is very high. Furthermore the low damping of its radiation permits of much simplification in the analysis of results. Several of the electrical measurements which may be done with such an oscillator have been described.[4]

Preliminary Experiments

Many experiments were made in order to determine the applicability of continuous waves to the two general methods of measuring refractive index but they cannot be described in detail here. The method used in this research involved the propagation of waves through the two media being compared. If standing waves are simultaneously produced in the two media the wave frequencies are the same, so that the index of refraction may be expressed either as a ratio of wave-lengths or a ratio of distances between voltage nodes. An investigation was therefore made into the lengths of standing waves as effected by, first, the resistance of the wires, second, the terminal conditions which determine reflection and third, the coupling between the oscillator and the resonator. The results of the investigation of the first of these points showed that, for the range of frequencies used, conductors having conductivity equal

[3] W. C. White, General Electric Rev., **19**, 1919
B. van der Pol, Phil. Mag. **38**, 1919
C. Gutton and M. Touly, Comptes Rendus, **168**, Feb. 3, 1919

[4] G. C. Southworth, Radio Review, **2**, Jan., 1921

to or greater than No. 18 copper wire caused little effect on the apparent wave-length. If however iron wire or high resistance alloy wires be used, the effect may be quite serious.

The second effect which may be very serious is that produced by imperfect reflection at the ends of the parallel wires. This of course may be eliminated by choosing wires of sufficient length so that several nodes and loops may be produced. The wave-length is then determined from distances between voltage nodes other than those at the ends. If the waves to be measured are fairly long this is usually inconvenient. Moreover the sharpness of resonance is not as marked as when the wires are short. If one end of the wires is left open, the distance from this end to the nearest voltage node would ordinarily be equal to a quarter wave-length, but this is not quite true. If the wires are bridged by a conductor, the distance to the nearest voltage node should be equal to half a wave-length, but this also is only an approximation unless certain precautions are observed.

The results of the preliminary experiments indicated that the correction to the open end line is approximately equivalent to adding the wire separation. The correction when the wires are closed by a conductor of small reflecting area is considerably different from that for large conductors. These results seem to indicate that the effect depends more on the shape and size than on the conductivity of the bridge. They furthermore agree with the predictions of Bumstead.[5]

Experiments were made to determine the effects caused by coupling the oscillator at different places along the wires. These showed slight differences which were hardly more than the experimental errors in determining resonance. The coupling in most cases was at the end of the resonator remote from the thermo-element. It was most convenient to place the plane of the oscillator perpendicular to the plane of the resonator. This was possible providing the electric force be kept parallel to the plane of the resonator.

These phenomena may be readily explained qualitatively if use is made of Faraday's conception of lines of electric and magnetic force as extended by Maxwell and others.

Measurement of Wave-Length in Water

The measurement of the length of waves in water is more difficult than for those in air both because of the lack of sharpness of resonance and the spurious effects caused by the form of the vessel. These are shown by the following experiments. In one case a parallel wire system

[5] H. A. Bumstead, American Journal of Science, **14**, Nov. 1902.

consisting of brass tubing 1.6 cm in diameter, spaced either 5 cm or 10 cm, was placed in a wooden tank 30×30×125 cm. The tank was filled with water and the nodes and loops found. The coupling was brought about by placing the oscillator on the outside of the tank near one end. The adjustable bridge used for locating current loops consisted of a plane of sheet copper, 30 cm square located at right angles to the wires. This plane was made in two halves connected by two wires of a cross wire thermo-element. The other two wires of the thermocouple led to a sensitive galvanometer of low resistance. Later it was found more feasible to locate the resonance points by noting changes in the plate current of the electron tube. This method was checked against that using the thermo-element.

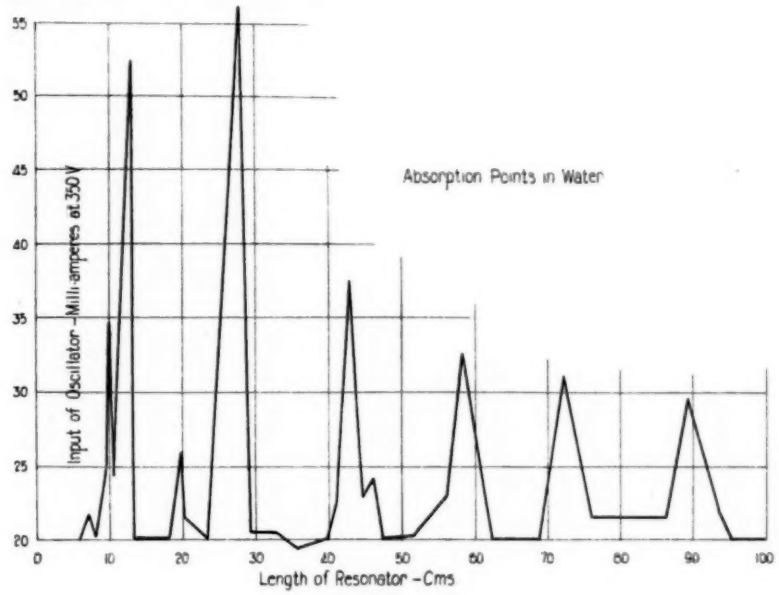

Fig. 2

The frequency of the oscillator was only slightly affected by this absorption of power, as is evinced by the concordance among the various values of wave-length as obtained from the plate current variations shown in Fig. 2. These variations are much greater than is usually observed for the resonance reaction on an ordinary electron tube circuit. This, coupled with the fact that the frequency is only slightly affected by the large current variations, would indicate that the phenomena were quite different in the two cases.

The absorption of radiation by the tank of water probably reduces the pressure reaction on the lines of force being propagated along the rec-

tangle and more of them become detached. The tank of water and the surrounding space may be considered as presenting an impedance toward the oscillator. When the Lecher system is in resonance this impedance is mostly resistance. Since the frequency is only slightly affected by this reaction, it follows that the coupling does not materially alter the effective inductance and capacity of the oscillator and that the load behaves as a pure resistance at all times. The curve shown in Fig. 2 consists mainly of straight lines connecting points of maximum and minimum deflection and therefore does not give a correct impression of the sharpness of resonance. Measurements made with tap water and a solution of copper sulphate indicated that the refractive index of water was only slightly affected by impurities.

It was found that the type of coupling described above led to curious results. Instead of being constant, the refractive index was found to vary from 8.7 at a wave-length of 111 cm to 4.6 for a wave-length of 258 cm. Tests made with tanks of various sizes suggested that this phenomenon was similar to that commonly assumed for the explanation of anomalous dispersion.

This effect was eliminated by extending the parallel wires through the end of the tank a distance of a half wave-length as measured in air. The oscillator was coupled to the free ends and their length adjusted so that the end of the trough became a voltage node. Measurements were taken as before. The results showed very little progressive error. This indicated that the error was due more to conditions in the wave front when it became attached to the wires than the proximity of the walls of the tank. Though this method eliminated the progressive error it was not as accurate as was desired because of the uncertainty in locating nodal points.

Final Experiments

The use of variations in the output of the oscillator as a means of locating nodes and loops and the use of conductors extended through the end of the tank suggested a method which proved to be capable of great accuracy. It is strikingly similar to Kundt's method for comparing sound velocities. The arrangement of the apparatus by which it was tested is shown schematically in Fig. 3.

A glass tube T, connected by means of flexible tubing to a cistern C contains the wires on which the wave-length in water is determined. The system on which the waves in air are measured is included between the stationary bridge B and the adjustable bridge B′. The stationary bridge contains a thermo-element for showing resonance. Its construc-

tion is shown in detail in Fig. 4. It was found that if, after adjusting the lower system to resonance, water was let into the tube, the deflection of the galvanometer which was connected to the bridge B was greatly

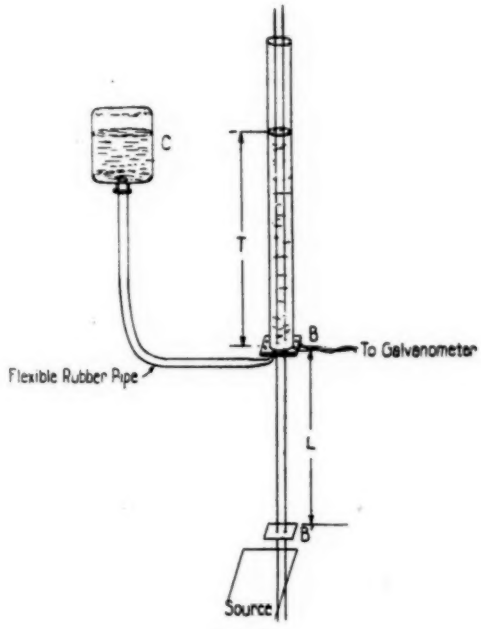

Fig. 3

reduced periodically as the water level passed over points which corresponded to voltage loops in the water. These points were quite sharply defined as is shown in the curve of Fig. 5.

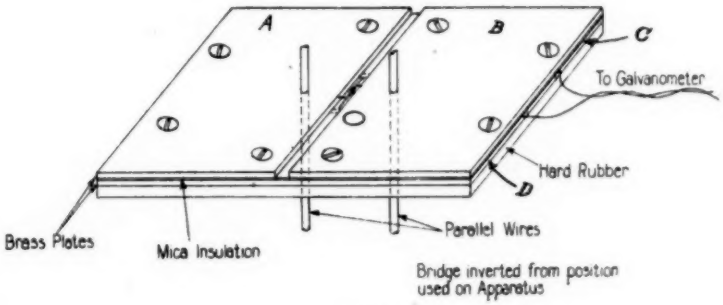

Fig. 4

A resonator of the type shown in Fig. 3 was constructed with the following dimensions: length of glass tube 115 cm, diameter of glass tube 7.5 cm, spacing of wires 2.5 cm, diameter of wires 0.1 cm. The

two reflecting bridges were approximately 12.5×12.5 cm. The lower one was sheet brass approximately 2 mm thick with spring clips to insure contact with the parallel wires. The upper bridge shown in Fig. 3 was so designed that only a small amount of the energy in the resonant system in air could pass into the system contained in glass tube.

The two plates A and B were in electrical contact with the parallel wires. The two plates C and D were placed at right angles to A and B and were insulated from them by mica except for the thermo-element. The bridge was mounted on a sheet of hard rubber 0.6 cm thick. The rubber mounting was bored to a depth of 0.4 cm in order that it might be more securely waxed to the glass tube. The thermo-element consisted

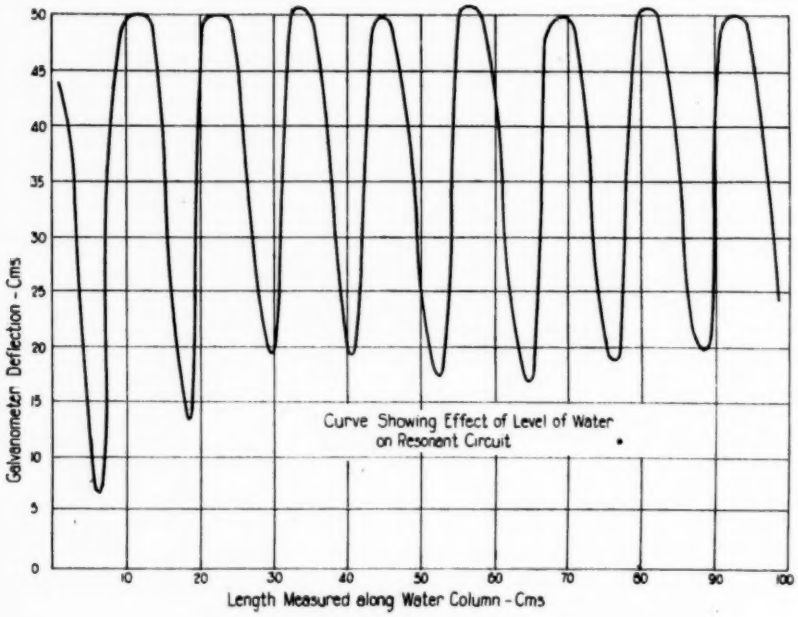

Fig. 5

of cross wires of constantan and steel approximately 0.006 mm in diameter. The sensitiveness of such an element varies somewhat with the manner in which it is mounted but one of the resonators used gave a deflection of 35 cm at a scale distance of 135 cm, for a low frequency current of 0.05 amperes. The resistance of the high frequency side of the element was 0.86 ohms and the resistance of the galvanometer was 3.5 ohms.

It may be observed that the coupling between the two systems can take place only through the opening in the stationary bridge. This area was made very small. The coupling effect was further reduced by the

presence of the thermo-element itself. Data obtained indicate that when the coupling is too great, slight progressive errors are produced.

Measurements were taken as follows: The lower bridge was adjusted for resonance without water in the tube, and the distance between the bridges noted. This distance L was taken as half of the wave-length for free space. While the system was still tuned to resonance water was forced into the tube either by elevating the cistern or by applying air pressure. The deflection of the galvanometer varied in a manner indicated by the curves of Fig. 5. In a few cases each absorption point was measured in order to make sure that they were uniformly spaced, but most of the measurements were taken by observing the second absorption point and later another point when the water was near the upper end of the tube. From the measured distance and the number of nodes passed over, the wave-length was determined. The ratio of the wave-length in air to that in water was taken as the index of refraction. The wave-length in air was checked occasionally to make sure that the oscillator did not vary in frequency. Practically no trouble was encountered from this source.

Results

Measurements were made with different variations of the apparatus described above. These variations consisted of different spacings of the parallel wires, glass tubes of different diameters and different degrees of coupling. It was found that the first two variations produced negligible results but that a coupling area 0.3×1.0 cm caused a progressive error of about three per cent in refractive index in the range of wave-lengths from 200 cm to 300 cm.

The apparatus used for obtaining the final data given in the accompanying table consisted of a pyrex glass tube 170 cm long and 10 cm in diameter, and had parallel wires 0.1 cm diameter spaced 1 cm apart. The coupling area was only 0.2×0.3 cm.

Table

Measurements taken at 20°C

Wave-length in free space	Location of first node measured	Location of last node measured	Wave-length in water	Index of Refraction
124.4 cm	10.1 cm	86.5 cm	13.88 cm	8.97
135.6	25.0	101.2	15.24	8.90
155.8	12.3	117.8	17.58	8.86
161.8	13.0	94.9	18.20	8.89
178.8	14.8	126.9	20.38	8.77
192.8	16.4	114.6	21.82	8.84
231.0	18.8	97.2	26.14	8.84
245.0	20.3	103.1	27.60	8.88
276.4	24.9	117.6	30.90	8.94
			Mean	8.88
			Mean deviation	±0.04

The results of the final experiments done in this research indicate that the absorption method of determining the position of nodes and loops of electrical waves in water is simple and accurate, but certain precautions must be observed regarding the coupling. The data obtained with the apparatus so designed as to minimize the errors, indicate that in the range of wave-lengths between 124 cm and 276 cm there is no appreciable dispersion for continuous waves. The mean value for the index of refraction of water was found to be 8.88. This is in close agreement with the results of the older investigators as well as the more recent work of Weichmann.[6] The results of this research do not agree with those of Blake[7] who obtained unusually low values for the dielectric constant. In view of the discrepancies in the preliminary experiments above, which were traced to the manner in which the apparatus was arranged, it is possible that the low values observed by Blake might be due to similar causes.

The agreement between the results of this research and those where damped waves were used show that within the limits of experimental accuracy there is no difference between the apparent velocity of groups of waves and of continuous waves. It is hoped that this work may be continued at frequencies at which absorption bands have been observed with the view of further testing the Sommerfeld-Brillouin Theory.[8]

SLOANE LABORATORY,
YALE UNIVERSITY,
November 15, 1923.

[6] Weichmann, Ann. der Phys. **66**, 501, 1921
[7] Blake, Phys. Rev. **15**, 148, 1920
[8] Sommerfeld and Brillouin, Ann. der Phys. **44**, 177-140, 1914

IMPROVED RADIOMETER CONSTRUCTION

By J. D. Tear

Abstract

Construction of more sensitive Nichols radiometer.—The sensitivity of the Nichols radiometer was increased by a factor of five by attaching "pearled" mica shields immediately behind the vanes. Optimum spacing of vane and shield was experimentally determined. Further increase in sensitivity was obtained by increased lightness of construction and by choice of dimensions and working conditions. Deflections were found proportional to energy, and the sensitivity per unit area proved independent of vane width for vanes less than 1.5×.5 mm. The most efficient fibre was found to be one which gives a half-period equal to the desired deflection time, and the best gas pressure that which gives critical damping. The sensitivity obtained is 2.1 (10)$^{-8}$ watts/cm^2 for 1mm deflection. Thin platinum films make good absorbing surfaces for radiometer vanes. *Absorption of long infra-red waves by thin platinum films* was found to reach a sharp and high maximum for a definite thickness, which, however, varies with the wave-length.

Introducton

DURING the course of recent investigations[1] in which an electric wave receiver embodying the radiometer principle was used, certain new features in radiometer construction were developed which gave promise of increased sensitivity if applied to the instrument in its classical form. Recently occasion has arisen to build a radiometer suspension with very small receiving surfaces for visible and infra-red radiation. Accordingly, a number of preliminary trials were made, after which the following construction was adopted.

Improved Construction

The suspension. The details of the suspension are shown in Fig. 1A which is drawn approximately to scale. The chief departure from earlier construction is the introduction of the shields *s* behind the vanes *v*. The shields and vanes are of mica which has been heat treated or "pearled." The mica strips after being cut to size are placed between two thin sheets of platinum which are then heated to incandescence. The effect of the temperature is to start innumerable cleavages within the mica, decreasing its thermal conductivity and incidentally producing a surface lustre similar to that of pearl. The vanes after being given a vanishingly thin coat of beeswax are blackened with camphor smoke. They are then

[1] E. F. Nichols and J. D. Tear, Phys. Rev. **21**, 587, 1923

fastened to the shields and properly spaced by means of a centrally placed fragment of quartz rod, 0.05 mm in diameter and 0.1 mm long, to which both are cemented with a small amount of fused shellac.

The cross arm and the axial quartz rod are about 0.02 mm in diameter. The platinum mirror which is chipped from a piece of optically plane glass is 1 × 1.3 mm with an average thickness of approximately 0.04 mm. The suspensions used in the present study with one exception weighed less than a milligram.

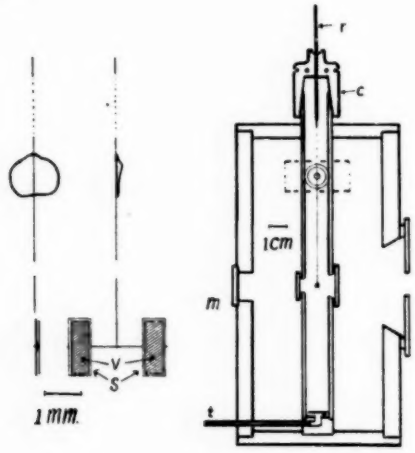

Fig. 1A Fig. 1B

Suspended system, showing relative position of vanes and shields. Radiometer case.

Separation of vanes and shields. The fact that the pressure corresponding to the maximum sensitivity of a simple radiometer is determined by the dimensions of the vanes suggests that the shields may introduce an auxiliary radiometer effect which would have a maximum for a given spacing of the vane and shield. The sensitivity of a system with vanes 1.5 × 0.5 mm, similar to that of Fig. 1, was tested with this in view. The relative sensitivities for four different vane and shield separations, the other constants of the system remaining unchanged, are given in Table I. As the greatest sensitivities were obtained with separations less than 0.1 mm, which is very small compared to the molecular mean free path at

TABLE I

Effect of varying distance between vane and shield

Separation of vane and shield	Relative sensitivity	
	Pressure 0.1 mm	Pressure 0.01 mm
0.07 mm	1.3	1.7
0.11	1.0	0.9
0.15	0.6	0.8
0.19	0.4	0.4

the pressures used, it is doubtful if the shield plays any part other than helping to maintain a large temperature difference between the exposed and the rear surface.

Vane width. In order to determine the relative sensitiveness of wide and narrow vanes, two suspensions were constructed which were sufficiently rugged to enable a series of vanes to be mounted in succession without the fibre being broken or the system otherwise changed. The receiving elements of the first system consisted of strips of clear mica blackened on one side and used as vanes without shields. The vanes were of the same length and thickness (1.5×0.015 mm) and ranged in width from 0.1 to 0.5 mm. The results appear in the upper half of Table II.

TABLE II

Ratio of deflections to area of exposed vane

Vane	Shield	Deflection	Relative sensitivity per unit area
1×0.1 mm	none	6 mm	60
1×0.3	none	18	60
1×0.5	none	28	56
1.5×0.002 mm	1.5×0.3 mm	.5	250
1.5×0.1	1.5×0.3	32.5	216
1.5×0.2	1.5×0.3	68	226
1.5×0.3	1.5×0.3	100	222

In the second system shielded vanes were used. The width of the vanes was varied from 0.3 mm to 0.1 mm, although the shields which were of clear mica (1.5×0.3 mm) were unchanged, and finally a bare platinum wire 0.002 mm in diameter was used as an extremely narrow vane. The shields alone gave a small positive deflection (0.5mm) which has been subtracted from the values given in the lower half of column 2 of Table II. The deflections recorded are for pressures corresponding to the maximum sensitivity of each vane. The exposures were made to the radiation from an incandescent lamp operated at constant current at a distance of 3 meters, giving at the radiometer an equivalent of approximately 0.1 candle-meter.

The procedure involved in removing and replacing the vanes of so delicate a system is not one which lends itself to precision. The data, however, are sufficiently accurate to show the deflections to be proportional to the vane width, i.e. that for such small vanes the sensitivity per unit area is independent of the width of the vane.

Lever arm. If the period is to remain constant while the distance of the vanes from the axis of rotation is varied, elementary considerations give a maximum of sensitivity for an undamped system when $r = \sqrt{I_0/m}$ in which I_0 is the moment of inertia of the rod, mirror and crossarm, m the mass of the vanes, and r the distance of the vanes from the axis.

Approximate calculations for the systems used give values of r ranging from 1 to 3 mm. In practice, however, the choice of the lever arm has been determined almost altogether by the damping. A very sensitive system was constructed with 0.1mm vanes and lever arms of 0.075 mm and an overall width of 0.25 mm. Such a system with vanes not compensating but so arranged as to produce a couple can be used in infrared spectrometry without introducing a second slit. However, because of its small damping, such a system is more sensitive to mechanical disturbances than are the larger systems. When the lever arm becomes 3 mm or greater, though the mechanical stability is good, there is an appreciable loss in sensitivity. A lever arm of from 1 to 2 mm has been found to be the most satisfactory compromise for 0.5×1.5 mm vanes.

The radiometer case. A sketch of the radiometer case is given in Fig. 1B. The cap with its ground joint at the top of the inner brass tube serves as a torsion head. There is a small axial steel rod r by means of which the height of the suspension can be adjusted. The windows are of optically plane glass and are cemented on with beeswax-resin mixture. The evacuation takes place through the 2 mm copper tube t, which, being flexible, facilitates the initial setting up of the vacuum system as well as subsequent changes in the position of the radiometer.

The second thick walled brass case serves as a protection aginst stray radiation and thermal disturbances. It has been found desirable to provide further protection by wrapping a thick layer of felt about the outer case. The case illustrated was used in earlier work with short electric waves in which a wide angle condensing lens was used, consequently the window apertures as shown are much larger than need be for work with visible or heat radiation. By means of a low power microscope placed at m, the image of the source may be brought into coincidence with the receiving vane.

A suspension with 0.5×1.5 mm vanes was used in this case and also in one of the type commonly used, described in detail by E. F. Nichols.[2] The difference in the sensitivity obtained with the two cases is not marked nor was there an appreciable change in sensitivity for these small vanes as they were moved from 2 mm to 6 mm from the window admitting the radiation he measured. The time required to secure a chosen degree of evacuation is much less, however, for the newer type of case. The original case was machined from a large brass casting, and due to the greater wall area and possibly to a somewhat porous structure of the casting, complete pressure equilibrium was never obtained until several hours

[2] E. F. Nichols, Astrophys. J. **13**, 101, 1901; Phys. Rev. (1) **4**, 297, 1897

after the pumping was stopped. With the newer case of commercial drawn brass tube no trouble has been experienced in securing and maintaining a chosen pressure: complete equilibrium is usually obtained well within an hour after stopping the pump.

Choice of fiber and degree of evacuation. It is possible to obtain quartz fibers which will give the radiometer a period of several minutes with the correspondingly great sensitiveness. Aside from the inconvenience of waiting so long for a deflection, there is a question whether such a system will give greater precision than can be obtained with the small deflections corresponding to a shorter period.

The condition of critical damping usually comes at a pressure well below that for the maximum radiometer effect; consequently the deflection time can be controlled by adjusting the pressure as well as by the choice of the fiber. Two pressure-sensitivity curves for suspension No. 4 (Table III) are shown in Fig. 2. The period was 14 seconds for curve *A* and 10 seconds for *B*. The solid lines represent steady deflections for long exposures and the dotted lines give the first turning points. *A'* and *B'* give the deflection times for each pressure for systems *A* and *B* respectively.

TABLE III

Relative sensitivities of various radiometers

Radiometer	Vanes	Period	Candle sensitivity*	Sensitivity per mm²†
(A) Nichols[2] 1901	2 mm diam.	13 sec.	724	136
(B) Porter[3] 1905	.6×6 mm	90	3200	11
(C) Coblentz[4] 1907	1×11	100	5700	5
(D) Coblentz[4] 1907	5×9	150	3200	3
(E) Rubens and Hollnagel[5] 1910 (Radiomicrometer)		20	1200	
(F) Witt[6] 1920 (Radiomicrometer)			1350	
(G) Weniger[7] 1910 (Bolometer)	.5×10.5	4	300	325
No. 1 Recent copy of (A)	2 mm diam.	13	470	89
No. 2 Same with shields	2 mm diam.	13	3430	650
No. 3	1×10 mm	38	14000	97
No. 4	.3×1.5	14	2100	2400
No. 5	.5×1.5	12	2600	2400
No. 6	.5×1.5	20	6200	2100

* Millimeters deflection for candle and scale each at one meter.

† Estimated deflections per square millimeter of exposed vane reduced to a ten second period.

The curves obtained of this type show that when a given suspension is used with different fibers, first, as may be expected, the deflections at a

[3] Porter, Astrophys. J. **22**, 229, 1905

[4] Coblentz, Bull. Bur. Stand. **4**, No. 3, 391, 1907–8

[5] Rubens and Hollnagel, Phil. Mag. (6), **19**, 764, 1910

[6] H. Witt, Phys. Zeit. **21**, 374, 1920

[7] W. Weniger, Phys. Rev. (1) **31**, 388, 1910

given pressure are proportional to the square of the periods, and second, that when the pressure is such that critical damping is exceeded the deflections are approximately proportional to the deflection times. Consequently, the most efficient fiber for a radiometer is one which will give a half-period equal to the largest permissible deflection time, and the pressure used should be such as to give critical damping.

When, however, an automatically timed shutter giving a relatively short exposure is used in connection with a fine fiber, pressures somewhat greater than those corresponding to critical damping give larger deflections and greater freedom from mechanical vibrations.

Blackening of vanes for the extreme infra-red. A thin film of platinum deposited upon mica by sputtering has been found to be a most effective absorber for the 320 μ radiation from the quartz mercury arc. The thick-

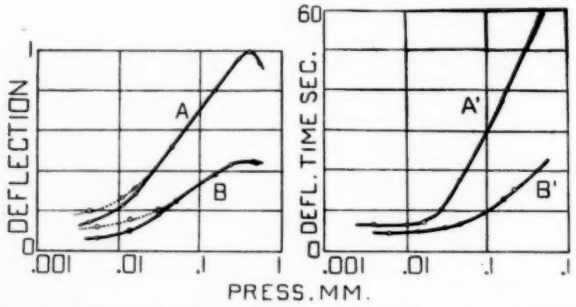

Fig. 2. A, B, pressure-sensitivity curves for same system with two different fibers; A′, B′, corresponding times for full deflection as pressure is varied.

ness of film for maximum absorption is a critical one and is evidently a function of the wave-length. The response of a radiometer vane to 320μ radiation as the thickness of the platinum film is varied, is shown graphically in Fig. 3. The thickness of the film is given in terms of its transmission for red light. The maximum for a transmission of 48 per cent is very sharp. A corresponding maximum for electric waves 6 mm long occurs at 8 per cent. It is very probable that this property of selective absorption of thin films can be carried to advantage far into the infra-red spectrum. However, for the total radiation from a candle or that from an incandescent lamp a platinum film on mica alone or as a foundation for camphor black was found less effective than camphor black alone.

Sensitivity

Although candle sensitivity is not an accurate measure, it is a convenient and about the only means of comparing the sensitivity of radio-

meters belonging to different epochs in the development of radiation measurements. For the purpose of comparison, brief specifications and candle sensitivities of some of the more sensitive radiometers and radiomicrometers, for which data are available, have been collected in Table III.[8] The values in the last column are estimated by assuming the deflections to be proportional to the period squared. Although this relation has been found to hold for No. 3 and No. 4, it may deal unfairly with

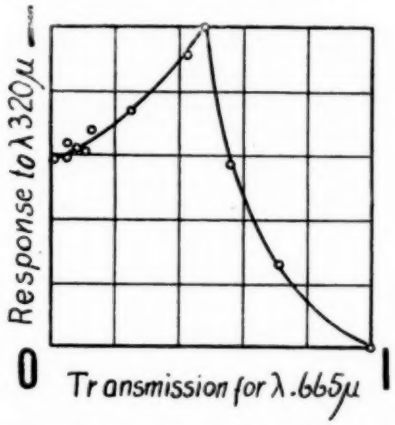

Fig. 3. Response of radiometer to constant 320μ radiation as thickness of platinum film is varied; thickness measured in terms of transmission for red light.

the longer period systems which were probably much more than critically damped. The numbers 1 to 6 refer to radiometers with shielded vanes as described in this paper. The sensitivities are for a paraffin candle burning 7.7 gm per hour.

TABLE IV

Proportionality of deflections to energy intensity

Sector opening	Photometric transmissions	Radiometer deflections (reduced)	Difference
360°	100.0	100.4	+.4
270°	74.9	75.0	+.1
180°	49.9	49.2	−.7
90°	24.4	24.1	−.3
30°	8.30	8.60	+.30
6°	1.64	1.60	−.04

Proportionality of deflections to intensity of radiation. The relation between deflections and energy intensity was determined for radiometer No. 6, Table III, by measuring the transmissions of a series of rotating sector disks. The sector disks had been previously calibrated photometrically

[8] B. J. Spence, (Am. Opt. Soc. J. **6**, 625, 1922) has obtained great freedom from zero drift by using blackened phosphor-bronze strips as vanes.

and the values of the transmissions are known to within one-half per cent.[9]

As may be seen in Table IV, the agreement between the photometric and radiometric values is sufficiently close to show the deflections to be quite strictly proportional to the energy intensity. The radiometer was used at highest sensitivity, 6200 scale divisions per candle-meter. An automatically timed shutter gave an exposure time of 10 seconds, and the pressure used, 0.012 mm, was such as to give somewhat more than critical damping. Under these conditions the probable error of the mean of 10 observations was 0.2 mm for a scale distance of one meter. With further precautions to insure thermal insulation this value can doubtless be considerably diminished.

From a calibration with a Hefner candle, taking 1.1 $(10)^{-4}$ watts /cm^2 as the intensity of the total radiation of the Hefner candle at one meter[10] and correcting for the absorption of the window, one millimeter deflection was found to correspond to an intensity of 2.1 $(10)^{-8}$ watts/cm^2. For the smallest deflection (.2 mm) which could be measured with certainty under the above conditions, energy was intercepted by the .5 $\times$1.5 mm vane at the rate of 3×10^{-4} ergs/sec.

Conclusion

Of the fifteen-fold increase in sensitivity which has been obtained, a factor of from 4 to 6 is due to the addition of shielded vanes of the type described. The remainder is accounted for by increased lightness of construction and the proper choice of dimensions and other working conditions. Shields should be placed as close behind the vanes as possible. The proportions of the vanes may be varied over a wide range without material change in sensitivity per unit area.

Nela Research Laboratories,
National Lamp Works,
Cleveland, Ohio,
October 25, 1923

[9] F. E. Cady, Trans. I. E. S., **16**, 138, 1921
[10] W. W. Coblentz, Bur. Stand. Sci. Pap. **11**, 87, 1915

ON METHODS OF VARYING THE SENSITIVENESS OF BALLISTIC GALVANOMETERS

By Morton Masius

Abstract

Methods of changing the sensitivity of ballistic galvanometers in a definite ratio by adding series resistance.—On closed circuit, the added series resistance does not change the throws in the inverse ratio of the circuit resistances because of the simultaneous change in electromagnetic damping. However, if the added resistance r_2 required to reduce a given throw to half value is determined by measurement so that $r_2/G=n_2$ (where G is the original total resistance of the galvanometer circuit) is known, then n_5 and n_{10} and the corresponding resistances r_5 and r_{10} required to reduce the throw to one-fifth and one-tenth, can be obtained from a table which is given. A slight amount of air damping does not affect the results, but for galvanometers with a special damping coil this table does not hold. The complication due to the change of the damping may, however, be readily eliminated if a special switch is used to open the galvanometer circuit immediately after the induced charge has passed. In this case even with strongly damped galvanometers, experiment shows that the deflections are inversely proportional to the total resistance of the galvanometer circuit.

WHEN a moving-coil galvanometer forms part of a closed circuit, the total resistance of which we denote by G, and an induced charge Q is made to pass through the circuit, a throw is observed which we shall call x. This throw is much smaller than the corresponding throw d_1 which would be observed if the same quantity Q were discharged through the same galvanometer from a condenser, because in the latter case the galvanometer (save for air damping) swings freely, while in the former case the e.m.f. induced in the galvanometer windings by the motion in the field of the field magnet opposes the motion. However, it can be easily shown experimentally and theoretically[1] that in spite of this electromagnetic damping, different throws x are proportional to the corresponding quantities Q, just as the undamped throws d_1 are.

If, as is sometimes the case (*e.g.*, in the measurement of magnetic induction), quantities Q of different order of magnitude have to be measured, it becomes desirable or even necessary to reduce the throws x corresponding to the larger quantities. If this is done by introducing an additional series resistance r in the galvanometer circuit and the throw obtained with the resistance r added be called y, then

$$x/y<(G+r)/G$$

[1] A brief summary of the theory of the moving-coil galvanometer is given e.g. in Kohlrausch, Lehrbuch der prakt. Phys. §§108, 109.

The throws x and y are *not* inversely proportional to the resistances, because, while the quantities originally induced in the galvanometer circuit are inversely proportional to the resistances, the throws x and y depend also on the damping, and the damping in the case of the throw y is always relatively less than in the case of x, because the additional resistance in the galvanometer circuit decreases the damping current.

Two methods will be given in the following that will enable the experimenter to *change the sensitiveness of his galvanometer at will in a known proportion by the addition of series resistance.*

First Method

Suppose a suitable throw x has been obtained with no extra resistance, the total resistance of the galvanometer circuit being G, and then a series resistance r is added and adjusted to a definite value r_2, such that y is just half of x $(x/y=2)$, then the ratio $n_2=r_2/G$ is a constant which is characteristic of this circuit. If we now call the series resistance that will reduce a given throw x to one-fifth $(x/y=5)$ r_5, and similarly one that will reduce x to one-tenth r_{10}, and put $n_5=r_5/G$ and $n_{10}=r_{10}/G$, the relations between n_2, n_5, n_{10} are represented by the following table.

n_2	n_5/n_2	n_{10}/n_2
1.200	3.994	8.987
1.400	3.981	8.966
1.600	3.970	8.912
1.800	3.965	8.896
2.000	3.956	8.870
2.200	3.946	8.839
2.400	3.932	8.806
2.600	3.920	8.774
2.800	3.911	8.745
3.000	3.902	8.720
3.500	3.882	8.665
4.000	3.864	8.615
4.500	3.847	8.565
5.000	3.831	8.520
6.000	3.808	8.459
7.500	3.773	8.366

The use of the table may be illustrated by an example. Suppose, *e.g.*, the total resistance G of the galvanometer itself and the secondary coil in series with it is found by direct measurement to be 125 ohms. A suitable throw is obtained by reversal of the current in the primary, and then a box resistance of 445 ohms in series is found to reduce the throw to half its value. This gives $n_2=445/125=3.560$. By interpolation in the three columns of the table we find

$$n_5/n_2=3.882-(60/500)18=3.880$$

and

$$n_{10}/n_2=8.665-(60/500)50=8.659.$$

Hence the series resistance required to reduce the throw to one-fifth would be

$$r_5 = G\, n_5 = 125 \times 3.560 \times 3.880 = 1727 \text{ ohms},$$

and the resistance required to reduce the throw to one-tenth would be

$$r_{10} = G\, n_{10} = 125 \times 3.560 \times 8.659 = 3853 \text{ ohms}.$$

These interpolations can easily be made with an error of no more than one unit in the fourth significant figure. If the observer desires to reduce the sensitiveness of his galvanometer not in the ratio 1/5 or 1/10 but in some other ratio, e.g., 1/7, he might interpolate between n_5 and n_{10} as follows:

$$n_5/n_2 = 3.880, \quad n_{10}/n_2 = 8.659,$$

hence

$$n_7/n_2 = 3.880 + (2/5)4.779 = 5.792.$$

These interpolations are not as accurate as the former but they will only rarely have to be performed.

As the following proofs will show, *this does not apply to galvanometers that are equipped with a special damping coil, but a slight amount of air damping has no appreciable effect on the results.*

An outline of the theory is as follows. The equation

$$d^2\theta/dt^2 + a\, d\theta/dt + b\theta = 0$$

is solved for θ and from the value of θ as a function of t the general expression[2] for $d\theta/dt$ is obtained, equated to zero, and solved for t. This particular value of t, say t_1, is substituted in the expression for θ, and this gives one of the following three expressions for θ_m, the maximum value of θ:

(a) $\theta_m = (\Omega/\sqrt{b})e^{-(aT/4\pi)\tan^{-1}(4\pi/aT)}$

(b) $\theta_m = \Omega/e\sqrt{b}$

(c) $\theta_m = \dfrac{\Omega}{2B}\left(\dfrac{A+B}{A-B}\right)^{-A/2B}\left[\left(\dfrac{A+B}{A-B}\right)^{\frac{1}{2}} - \left(\dfrac{A+B}{A-B}\right)^{-\frac{1}{2}}\right]$

where Ω is the initial angular velocity; the period T is given by

$$T = 2\pi/\sqrt{b - a^2/4},$$

and A and B are given by

$$A = \tfrac{1}{2}a, \quad B = \sqrt{a^2/4 - b}.$$

The first solution (a) applies to the case of a galvanometer for which

$$b > a^2/4$$

or, if we write

$$a^2/4 = mb,$$

for which

$$m < 1,$$

while for the case $m = 1$ (critical damping) we get solution (b), and for strongly damped galvanometers ($m > 1$) we get solution (c).

[2] See Kohlrausch, l.c.[1]

The throws x and y read on the scale are, of course, proportional to the angles θ_m, and hence the ratios x/y are obtained by the ratio of expressions of the types (a), (b), or (c).

Five cases may arise in practice, viz.,

(1) x and y are both of type (a);

(2) x is of type (b) and y of type (a);

(3) x is of type (c) and y of type (a);

(4) x is of type (c) and y of type (b);

(5) x and y are both of type (c).

In performing the calculations, we may further note that

$$\Omega_x/\Omega_y = (r+G)/G = n+1,$$

since the initial velocities are proportional to the original discharges passing.

We also have

$$a_x^2/4 = m_x b$$
$$a_y^2/4 = m_y b,$$

b, which depends on the torsional elasticity of the galvanometer suspension, being the same, no matter whether resistance is added or not. Furthermore

$$a_x/a_y = (G+r)/G = n+1$$

because a, which depends on the damping current, is inversely proportional to the resistance of the galvanometer circuit. If all these substitutions are carried out, and the resulting expressions reduced to a simple form, we get for the calculation of x/y in the five cases:

$$\frac{x}{y} = (n+1)\frac{e^{\sqrt{m_x/[(n+1)^2-m_x]}\,\tan^{-1}\sqrt{[(n+1)^2-m_x]/m_x}}}{e^{\sqrt{m_x/(1-m_x)}\,\tan^{-1}\sqrt{(1-m_x)/m_x}}} \tag{1}$$

$$\frac{x}{y} = (n+1)\,\frac{e^{1/\sqrt{(n+1)^2-1}\,\tan^{-1}\sqrt{(n+1)^2-1}}}{e} \tag{2}$$

$$\frac{x}{y} = (n+1)\frac{e^{\sqrt{m_x/[(n+1)^2-m_x]}\,\tan^{-1}\sqrt{[(n+1)^2-m_x]/m_x}}}{[\sqrt{m_x-1}+\sqrt{m_x}]^{\sqrt{m_x/(m_x-1)}}} \tag{3}$$

$$\frac{x}{y} = (n+1)\frac{e}{[\sqrt{m_x-1}+\sqrt{m_x}]^{\sqrt{m_x/(m_x-1)}}} \tag{4}$$

$$\frac{x}{y} = (n+1)^2\,\frac{[\sqrt{m_x-(n+1)^2}+\sqrt{m_x}]^{\sqrt{m_x/[m_x-(n+1)^2]}}}{[\sqrt{m_x-1}+\sqrt{m_x}]^{\sqrt{m_x/(m_x-1)}}} \tag{5}$$

These five relations represent x/y as a function of two parameters, viz., m_x, which, since it depends on b and a_x, is a constant characteristic of the original circuit only, and n, which is the ratio of the added resistance to the original resistance. The calculations give originally x/y as a function of these two parameters, but since m_x, for strongly damped galvanometers at least, is not a parameter that can be readily measured

for a given galvanometer circuit, a new table was deduced from the original one in terms of the quantities n_2, n_5, and n_{10} used above.

If the damping is not entirely electromagnetic but partly due to air, we have in addition to a term a, which varies inversely as the resistance, a constant small term representing air damping, say a_a. Thus

$$\frac{a_x}{a_y}=\frac{a_a+k/G}{a_a+k/(n+1)G}.$$

This may be written in the form

$$\frac{a_x}{a_y}=(n+1)\ \frac{(1+a_aG/k)}{(1+a_aG(n+1)/k)}.$$

If we now assume that air damping is slight, then

$$a_a << k/G,$$

and hence we get approximately (for not too large values of n)

$$a_x/a_y=(n+1)\ (1-n\ a_aG/k)$$

instead of $a_x/a_y=(n+1)$ as used in the calculations. To form an estimate of the error involved by the omission of air damping we may calculate the contribution that a_a makes to a_x in a definite case.

It can be easily shown that the quantity m defined above is related to the logarithmic decrement λ by the relation

$$m=\lambda^2/(\pi^2+\lambda^2).$$

If we now assume the vibrations, damped by air alone, to decrease with a damping ratio of 40/39 (which seems a sufficient allowance for modern moving-coil galvanometers in which air damping is small) we find

$$m=61\times10^{-6}$$

and

$$a_a=2\sqrt{b}\ \sqrt{m}=2\sqrt{b}\times8\times10^{-3},$$

while the smallest value of a_x considered (the case of $n_2=1.200$) corresponds to a value of m of about 0.031 and hence to

$$a_x=2\sqrt{b}\times18\times10^{-2}.$$

For the larger values of m, e.g., 1.5 corresponding about to $n_2=3.5$, the air damping becomes a quite insignificant fraction of the total damping.

The ratios x/y obtained from the table were verified experimentally for several galvanometers of very different damping and the experimental results agreed with the calculated values within the limits of experimental error.

B. O. Peirce[3] published results which seemed to show, as a purely empirical result, that there exists a *constant* quantity B, which might be suitably called the *effective resistance of a ballistic galvanometer on closed circuit*, such that

$$x/y=(B+r)/B$$

[3] B. O. Peirce, Proc. Am. Acad. of Sci. **42**, p. 161 (1906)

or, $$B = r\ y/(x-y).$$

Using the notation previously adopted we find

$$B = G\ n/(x/y-1).$$

Our investigation shows that *B is not quite constant*. If it were, the column n_5/n_2 of our table would remain exactly 4, and the column n_{10}/n_2 would be exactly 9.

Second Method

A second method of avoiding the complications arising from the simultaneous decrease of the induced charge and the damping upon addition of series resistance is to arrange the galvanometer circuit so that it remains closed while the induced charge passes, but thereafter is opened and allows the galvanometer to swing freely on open circuit.

A crude method of accomplishing this is to connect both the primary and the secondary through an ordinary double-pole single-throw switch, the contacts of which have been cut off a little on one pole. If the primary is connected through this side and the secondary through the other, a swift motion throwing the switch off will break first the primary and then the secondary.

Much more precise results are obtained by a special switch constructed by the writer for permeability work. The contacts for both primary and secondary are made through springs leading to three partly split rings mounted on a cylindrical shaft. The exact position can be adjusted by a rotation about the shaft. This readily permits an arrangement for breaking the secondary circuit immediately after either break, or make, or reversal of the primary. The rotation of the shaft is made by the action of a strong spring, which the observer merely releases. There should be a slight correction to these open circuit throws for air damping and for the short time that the galvanometer moves with the secondary on closed circuit, but experiment indicates that if d_1 is the throw with the resistance G in the galvanometer circuit, and d_2 the throw with the added resistance r, $$d_1/d_2 = (G+r)/G.$$

Experimenting with several strongly damped Leeds and Northrup galvanometers ($m_\perp$ ranging from 1.5 to nearly 7), the writer found values for G calculated from

$$G = r\ d_2/(d_1-d_2)$$

that agreed with the resistance of the galvanometer directly measured, within the possible error determined by the uncertainty of the reading of the throws d_1 and d_2.

Worcester Polytechnic Institute,
November 29, 1923

A METHOD OF DRAWING METALLIC FILAMENTS AND A DISCUSSION OF THEIR PROPERTIES AND USES

By G. F. Taylor

Abstract

If a glass tube filled with a molten metal is placed in a tubular furnace or in a transverse hole through a heated copper rod and drawn out at the proper rate, metal filaments of almost any degree of fineness may be obtained down to a diameter of 10^{-5} cm, or even less. The glass or quartz used must soften at a temperature between the melting and the boiling point of the particular metal. Filaments of Pb, Sb, Bi, Au, Ag, Cu, Fe, Sn, Tl, Cd, Co, Ga, and In have been made by this method. The glass envelop may serve as insulation or may be removed, if desired, with HF. The filaments are very pliable and have greater *tensile strength* than wires of ordinary size. The *temperature coefficients of resistance* were not found to differ markedly from those of the metals in bulk, and to be more constant. Metallic filaments made in this way have been *used for resistance thermometers, thermocouples, galvanometer suspensions, and hair lines for the eye pieces of telescopes. Their use in micropiles, bolometers and the moving coil of both direct and alternating current galvanometers is suggested. Conducting quartz threads* were made by drawing down a tube with a silver core.

CERTAIN experiments conducted by the Bureau of Plant Industry which required thermocouples and resistance thermometers of exceptionally small dimensions have lead to the development of a process of drawing wire of extreme fineness, the description of which may be of interest in other fields.

The metals Pb, Sb, Bi, Sn, Cd, Ga, Tl, Cu, Ag, Au, Fe, In, Al and numerous alloys of these metals with each other were drawn into wire by the process described below, which applies in general to all these metals and alloys, though special comment must be made in most cases.

The process consists essentially of filling a glass tube with the metal from which the wire is to be drawn, placing this in a heated cylinder or flame and drawing it out as fine as desired. The glass selected must soften at a temperature between the melting and the boiling point of the metal used and must not react chemically with the metal at high temperatures.

A freshly drawn glass tube about 2 mm inside diameter is closed at one end. A bit of the metal to be drawn is dropped into the tube and lodges at the closed end. This end is heated in a flame until the metal

melts and the glass softens. Most of the metals form an oxide at this temperature but the oxygen near the metal is soon exhausted. The metal is freed from its coat of oxide by pressing the walls of the tube together with small forceps, causing the bright liquid metal to flow back to a clean portion of the tube and leave the slag behind. The original end of the tube, now containing only the slag, is drawn off and discarded. It may be necessary to repeat this operation a number of times for unless the metal appears as a flawless mirror against the inner surface of the glass, it cannot be drawn into a perfect wire. The new end, now containing the bright metal, is strongly heated and drawn out to a diameter of .5 to 1 mm and a foot or more in length. These metal filled glass tubes are now ready to be drawn through a heated cylinder to almost any degree of fineness.

The cylinder, or muffle, is made from a copper rod, 1 cm in diameter and 7 cm long (Fig. 1, a). Toward one end is a row of holes *f*, 2 mm in diameter, parallel to each other and perpendicular to the axis of the rod. These holes are each lined with a spiral of nichrome wire to prevent the semiliquid glass from adhering to the red hot copper. This rod is supported in a horizontal position by a smaller steel rod *c*, attached at one end to the copper rod, and at the other end to any kind of support.

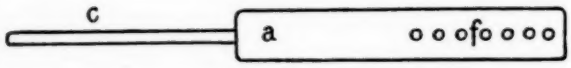

Fig. 1

By means of a flame the rod is heated to a bright red on the end nearest the support, the temperature diminishing toward the other end. The hole having a suitable temperature is selected and one end of the glass-covered wire, already prepared, is passed through the hole, grasped with small forceps and drawn out as fine as desired.

The size of the wire is determined by the wall thickness of the glass and the rate at which it is fed into and drawn out from the heated cylinder. When a constant rate is maintained, a wire of uniform thickness is drawn. By varying the ratio of the feeding in to the drawing out, a wire of any degree of taper within the limits of the original diameter may be produced.

When heated in a flame and drawn in air, those metals which contract on solidifying will form breaks at short intervals, while those which expand will rupture the glass capillary. When drawn slowly from a heated cylinder, however, a perfect cast is formed, because solidification takes place at a point adjacent to the liquid metal.

Special glasses must be used for most of the metals and the manipulation varies more or less with each case. Pb, Sn and Bi can be drawn in almost any kind of glass by the process described above. Tl may be drawn in the same way, but only in a certain borosilicate glass No. G-80 manufactured by the Corning Glass Works. Cd and Ga were drawn in a very soft glass No. G-1. Sb works best in a Jena glass No. 59[111] and the walls should be made as thin as possible. Au, Ag, Fe, and Co were drawn in quartz. In this case the muffle was made of a carborundum cylinder and heated by three oxy-acetylene flames focused upon it. An arm's length of Ag and shorter lengths of Au and Cu may be drawn directly from a flame in a very hard Corning glass No. G-707-CR.[1] In the case of Ag a borax bead should be dropped into the tube and fused with the metal. These metals and also Mn and Cr could probably be drawn to indefinite lengths in a glass which softened between 1200° and 1400°C. At present no glass is obtainable between quartz (about 1800°) and G-707-CR, (about 900°). Al combines with silicate glasses at the temperature of its melting point but was drawn in a tube made of its own borate. Fine threads of In were drawn in a very soft glass of unknown origin.

The glass may be removed from the metallic filament with hydrofluoric acid, which does not readily attack any of the metals which were drawn in silicate glasses except Ga.

Temporary electrical connections were made to filaments of the order .0001 cm and smaller by pressing the ends between bits of tin amalgam. Permanent connections were made to larger filaments by embedding them in alloys of suitable properties. One end of the filament was soldered to the body of the alloy and the other end to a wire which was insulated from the alloy by a glass tube.

Properties

Wires drawn in glass have brilliant reflecting surfaces. Owing to this and the principle of irradiation, wires as small as .0001 cm diam. are visible under the proper light conditions. Wires of this degree of fineness float in air as a single line of silk or spider's web and no means have been found for weighing them directly. However, the weight and diameter may be calculated from the resistance of a known length if we assume that the specific resistance of the filament is the same as that of the metal in bulk. One filament of Sb had a resistance of 125,000 ohms

[1] This special glass was obtained from the General Electric Company through the kindness of Mr. D. A. Mullaney.

per cm. Since the specific resistance of this metal has been determined[2] a simple calculation gives for this filament a mass of .000002 mg per cm and a diam. of .00002 cm (No. 92 B and S gauge).

On account of the importance of Bi and Sb in thermo-electric measurements, the physical properties of the very fine wire made from these metals deserve special attention. Though very brittle in bulk, Bi and Sb are pliable in the form of fine wire. Antimony wire as large as .003 cm diam. may be bent repeatedly without breaking. It is highly elastic and has a tensile strength of 1800 to 2200 kg per cm². In diameters .001 cm and smaller it will stand an indefinite amount of bending and twisting. Bi wire as large as .035 cm diam. when freshly made is very soft, less elastic than Pb and has a tensile strength of about 50 kg per cm². It rapidly hardens with bending and breaks after being bent into a right angle twenty times or so. The smaller sizes have greater tensile strength and the extremely small sizes, .002 cm and less, will stand indefinite bending.

No limit has been found to the smallness of filaments which can be produced. Even when they become invisible under the microscope their existence can be detected by making electrical contact to a source of current and a sensitive galvanometer. Boys[3] in his classical experiment with quartz fibers, estimated that a bit of quartz the size of a grain of sand, that is 1/60 mm³ if drawn into the finest thread would be 2000 miles long. As this fiber can be drawn with a gold core having a diameter only a small fraction of the total, it is clear that we can not assign any limit to the smallness of the gold fiber.

Uses and Suggested Uses

The most important application of these filaments has been in resistance thermometers made by embedding glass-insulated filaments in metals or alloys having suitable properties. Type metal was the alloy found to be most suitable for Pb filaments. More than one hundred of these thermometers were made, some of Pb and some of Tl. They were made in resistances covering a range from 10 to 5000 ohms and many of them have been kept under almost daily observation for several months. The lengths of the measuring elements were .5 to 1.5 cm and the diameter of the embedding alloy was about 2 mm. Those having the highest resistance showed a constancy of about 1 part in 15000 while those having resistances less than 100 ohms changed about 1/5000 of their total resistance in the same time. A better constancy may be

[2] Bureau of Standards, Circular 74, 1918

[3] C. V. Boys, Smithsonian Report 1890, p. 315

expected after a longer period of seasoning. Twenty-five of these Pb thermometers with resistances of 443.3 ohms are now in regular use by the Bureau of Plant Industry. With this resistance a change of 1 ohm is equal to 1°F within 1° over the range 22° to 38°. These thermometers will be more fully described in another article.

A convenient means of preparing wire in the laboratory for determining the temperature coefficient of resistance and hence the purity of a particular sample of metal, is afforded by this method of drawing wire in glass. It is especially convenient in cases where the quantity of metal is too small to be extruded into wire or analyzed chemically. Only a few mg are necessary to produce a satisfactory conductor. The method might be used to advantage in determining the electrical constants of the very rare metals provided their properties are suitable for drawing in glass.

Quartz fibers were made conducting by drawing them with a silver core instead of employing the former method of platinizing, silvering or gilding them on the outside. In fact it would be possible to make them doubly conducting by using both methods on the same filament.

Wire drawn in glass obviously is not contaminated by magnetic impurities as is wire drawn through steel dies. It seems possible, therefore, to eliminate zero shift[4] in moving coil galvanometers by winding the coil with wire drawn in glass and using a quartz suspension.

Hair lines for the eye pieces of telescopes may readily be prepared in the laboratory by drawing a fine thread of one of the metals or alloys and dissolving away the glass. Being opaque they make better defined lines than quartz or spider's web.

Thermocouples were made of extremely fine Bi and Sb wire. On account of the high resistance of these metals, the thermocouples must either be made very short or built up of cables of the filaments. Though this type of thermocouple is not practicable for ordinary use, except in special cases, it appears that it may be used to advantage in very sensitive micropiles and in instruments such as the Boys radiomicrometer and the Dudell alternating current galvanometer in which an infinitesmal heat capacity and lag are essential.

The nearest approach to this method of drawing fine wire, so far as the writer has been able to find, was that of W. H. Wollaston who, a century ago, drew Pt to a diameter of .000075 cm by using a silver wire with a Pt core, drawing both down and dissolving away the silver with nitric acid.

[4] W. P. White, Phys. Rev. **30,** 782 (1910)

THEORETICAL

If the large changes in resistance which are observed in the same pure metal after different kinds of heat treatment, strain etc. are due to variations in the size of the crystals, as has been suggested[5] we would expect filaments of minute diameter to show a greater constancy than the same metal in bulk, since, in case of the filament, a limit is set to the size of the crystals which can form. This is suggested as a possible explanation of the greater constancy of the higher resistance thermometers.

BUREAU OF PLANT INDUSTRY,
U. S. DEPARTMENT OF AGRICULTURE,
WASHINGTON, D. C.
December 10, 1923.

[5] P. W. Bridgman, Proc. Amer. Acad. vol. 52.

BOOK REVIEWS

General Physics for Colleges. D. L. WEBSTER, Stanford University, H. W. FARWELL, Columbia University, and E. R. DREW, Stanford University.—This textbook is both original and daring. The style is unconventional, and from the introductory remarks at the beginning of each chapter to the practical problems at the end, a strong appeal is made to the reader's acquaintance with everyday experience. Many topics usually considered essential in a college text have been omitted, in order to make room for the very excellent chapters at the end on "modern physics." Whatever doubt there may be as to the wisdom of introducing such subjects as the quantum theory and relativity in a first-year course, it cannot be denied that the authors have given a capital summary of these and of other important recent advances. The intelligent student who uses the book is pretty certain to become convinced that physics is interesting and related intimately to everyday life, and that many phenomena and laws in widely separated branches can be traced to the same fundamental principles. Nevertheless somehow we fear that he will feel that he is not "getting it" in as orderly and balanced a fashion as he would like. Whether this is a demerit in the book or one of its chief merits depends largely on the type of both student and teacher. The serious minded student who wants, as the youthful Maxwell said, to learn "how it doos," is frequently compelled to do one of two things, either of which is, to be sure, for the good of his soul—undertake some hard thinking, or else look the matter up in some other book. That this is just what the authors anticipated may be seen from the following quotation from the preface: "Permanent impr.ssions upon the mind of the student are determined largely by the strength and duration of the mental effort that he himse.f considers the subjects to be worth." The student is never allowed to lose sight of the experimental foundations of the science. It is regrettable, however, that so little attention is paid to the historical side of the subject, and that so few suggestions for collateral reading are given.

In the chapters on mechanics, force rather than mass is treated as a fundamental concept, so that the notation throughout this portion of the book is that of the engineer. Without wishing to revive an ancient controversy, the reviewer feels compelled to say that a trial of this method has not convinced him that the fundamentals of mechanics are made any clearer to the beginner by this method. Throughout the remainder of the book, mass or inertia is practically everywhere re-enthroned, as is inevitable when the subject matter is physics and not engineering.

Considering the importance that is attached to the understanding of everyday phenomena, it is curious that in the section on soap-bubbles no hint is given as to *why* a soap solution is used.

Diagrams and reproductions of photographs are excellent though some of the cuts have been too much reduced in size, and the English is particularly good. The derivation of the word "adiabatic" is, however, erroneous.—Pp. 557+XIV, 383 figures, and index. Century Co., 1923.

W. G. CADY.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY

MINUTES OF THE BERKELEY MEETING, MARCH 1, 1924.

The 126th regular meeting of the American Physical Society was held at the University of California at Berkeley, California, on Saturday, March 1, 1924. The presiding officer was Professor E. E. Hall.

The program consisted of fifteen papers, abstracts of which are given in the following pages. An **Author's Index** will be found at the end.

D. L. WEBSTER,
Local Secretary,
Pacific Coast Section.

ABSTRACTS

1. X-ray reflection from very thin crystals. O. L. SPONSLER, University of California, Southern Branch.—In a study of the structure of vegetable fibers, such as those from hemp, cotton, etc., a bit of direct evidence was found which favored the theoretical explanation of x-ray spectra as based on the wave theory. According to the theory many planes of atoms are necessary to produce a line with sharply marked edges and conversely a relatively small number of planes would produce a line with blurred edges. The fibers produce these two types of lines. When placed at right angles to a flat beam a clean cut line is produced. The planes producing this line are spaced 2.58 A apart. A millimeter of each fiber is across the path of the beam; this brings about 3,000,000 or 4,000,000 planes into position for effective reflection and the line produced agrees with the theory in that it has sharply marked edges. When the beam passes through the fibers lengthwise, however, blurred lines are produced. The planes are 6.10 A apart. With the end of the fiber presented to the beam only a very small portion is brought into an effective reflecting position, a portion only .003 mm thick containing a maximum of 5000 planes. Actually there are probably less than half this number effective because the fiber is decidedly rounded. The lines produced are much blurred, agreeing again with the theory.

2. An x-ray spectrograph for scattered radiation. P. A. ROSS, Stanford University.—An x-ray spectrograph has been designed which seems to be especially well adapted to work with faint scattered and fluorescent radiations. It combines the speed of the fixed crystal with the spectral range and simple relation of angle to wave-length of the rotating crystal. A fixed crystal has a lead or gold wedge of 60° angle brought into contact with the crystal face. No other slit is used. The scattering substance in the form of a plate or sheet is placed in front of the crystal and close to it. Scattered or fluorescent radiation coming from the plate will be reflected only if it strikes the crystal at the line of contact with the wedge at the correct angle of incidence. A small radiator near the crystal may thus cover several orders and give enough intensity for good photographs in a few hours.

3. A possible explanation of tertiary line spectra in x-rays. DAVID L. WEBSTER, Stanford University.—Clark and Duane (Proc. Nat. Acad., **10**, 41, 1924) report lines in scattered x-ray spectra, explaining them as continuous spectra excited by photo-electrons, with short-wave limits given by de Broglie's photo-electric energies. A serious objection to this explanation, noted by Compton, is that these rays are much stronger than would be expected; another that their spectra are very narrow, as contrasted with the wide continuous spectra of x-rays excited by cathode rays. The latter contrast Clark and Duane ascribe to filtering, but quantitative calculations prove filtering quite insufficient. The high intensity demands an unprecedented excitation efficiency, which can occur only in atoms in which the photo-electrons differ from ordinary cathode rays, namely in the atoms ejecting them. This means a very unusual probability of radiating impacts within these atoms. Since there is no previous penetration of other atoms, the spectra should resemble those of extremely thin targets, with reasonable filtering corrections. Such spectra may be predicted from the author's continuous-spectrum theory, (Phys. Rev. **9**, 220, 1917; **21**, 312, 1923) and agree with these qualitative data very much better than any thick-target spectra. This theory does not cover the Compton-Ross effect.

4. A graphical method for the utilization of rotation spectra in crystal structure determinations. MAURICE L. HUGGINS, National Research Fellow, California Institute of Technology.—For a crystal having rectangular axes, the l axis being in the axis of rotation, the coordinates of the nth order reflection from a crystal plane hkl on a photographic plate at a distance s from the crystal are:

$$x = s \tan 2\left\{\sin^{-1}\left[(n\lambda/2)\,(h^2/a^2+k^2/b^2+l^2/c^2) \div \sqrt{(h^2/a^2+k^2/b^2)}\right]\right\};$$

$$y = \sqrt{(x^2+s^2)} \tan\left[\sin^{-1}(n\lambda l/c)\right].$$

The distance from the central spot is $s \tan 2[\sin^{-1}\{n\lambda/2\sqrt{(h^2/a^2+k^2/b^2+l^2/c^2)}\}]$. With the help of a specially designed ruler, pivoted at the central image (of a print of the photograph) a projection is made in which the abscissas are proportional to $n\sqrt{(h^2/a^2+k^2/b^2)}$ and the ordinates are proportional to nl/c. The unit distances a, b and c are the smallest distances for which h, k, l and n will in every instance be integral provided only that there is no common divisor of *all* the h's or *all* the k's or *all* the l's of the planes for which reflections appear. To insure this the angle of rotation should be large enough for the crystal to be in position for reflection from at least one plane of every form, and the exposure should be sufficiently long. With h, k, l and n known for each spot, the analysis may proceed much as in the case of Laue photographs. The methods are similar with crystals having non-rectangular axes, but the formulas are more complicated.

5. The Duane-Compton quantum theory of diffraction. P. EHRENFEST AND P. S. EPSTEIN, California Institute of Technology.—The question is raised whether it is possible to extend the Duane-Compton theory to the case of finite gratings and of other diffracting structures. It is shown that this can be done by a procedure similar to Bohr's principle of correspondence. This procedure permits a complete treatment of all the Fraunhofer diffraction phenomena on the basis of the quantum theory with results in all cases identical with those of the classical theory. A similar treatment is given to the phenomena of Fresnel diffraction. The agreement with the classical theory, though far reaching, is here not quite so complete as in the Fraunhofer case. There are some indications that it is possible to construct a satisfactory classical analogy to Compton's diffraction even in the case of a change of frequency.

6. The fine structure of nitrogen, oxygen, and fluorine lines in the extreme ultra-violet. I. S. BOWEN AND R. A. MILLIKAN, California Institute of Technology.—In the endeavor to increase the resolution obtained in the analysis of vacuum spark lines in the extreme ultra-violet, spectra of orders up to the 7th have been used, and the structures of

all the strong nitrogen, oxygen, and fluorine lines between 500 A and 1500 A have been found to be as follows. (1) *Nitrogen lines.* λ685 and λ1085 are both quadruplets, 916 a triplet, and 991 and 1492 doublets. (2) *Oxygen lines.* λ834 is a septuplet, 554 is a quadruplet, 507, 610, and 703 are all triplets, the first and the third of these having identical frequency differences; 525, 599, 629, 644, and 718 are all singlets. (3) *Fluorine lines.* λ606 and λ656 are probably quintuplets. The wave-lengths of all the components of most of the foregoing lines have been fixed with an error of but about .02 A. This establishes a new set of standard wave-lengths in the extreme ultra-violet region between 500 A and 800 A.

7. The series spectra of the stripped boron atom (B_{III}). I. S. BOWEN AND R. A. MILLIKAN, California Institute of Technology.—The following B_{III} series lines have been located in spectra of order 2 to 6 with the accuracy of about .02 A and assigned to the series indicated. $(2p-3d)$ 677.01 and 677.16 I. A. (vac.); $(2p-3s)$ 758 47 and 758.68; $(2s-2p)$ 2066.41 and 2067.88; $(3d-4f)$ 2077.79. From these measurements the following term-values have been obtained: $4f=61750$; $3d=109878$; $2p_1=257553$; $2p_2=257587$, $2s=305946$; $3s=125744$. Two B_I doublets have been measured at 1825.87 and 1826.41 A and at 2089.60 and 2090.29 A.

8. Duration of molecules in upper quantum states. RICHARD C. TOLMAN, California Institute of Technology.—Making use of Einstein's views as to the mechanism of the absorption, emission, and negative absorption of light by molecules, together with Füchtbauer's method of determining the probability of absorption, the following equation is derived:

$$A_{21}=(8\pi\nu^2/c^2N_1)\ (p_1/p_2)\int_0^\infty a\,d\nu$$

where A_{21} is the chance per unit time that a molecule in quantum state 2 will spontaneously jump to the lower state 1, ν is the frequency of the light emitted in such a jump, p_1 and p_2 are the a priori probabilities of states 1 and 2, and a is the coefficient of absorption for light of frequency ν when N_1 is the concentration of molecules in quantum state 1. In case the jump to state 1 is the only one possible, the mean life τ of the molecule in quantum state 2 is the reciprocal of A_{21}. Values of A_{21} and τ are calculated from existing data for a considerable number of quantum jumps and their significance discussed.

9. Velocity distribution of secondary electrons. JOSEPH A. BECKER, National Research Fellow, California Institute of Technology.—Electrons, accelerated by 200 volts, strike a target. Some secondary electrons strike another target and are then analysed by means of a magnetic field. The results show a relatively intense narrow band with sharp edges which correspond to within ½ per cent to the voltage from the ends of the filament to the target. Then follows a region of practically no blackening, whose width corresponds to 4 to 11 volts depending upon the targets. Next comes a hazy edged band which extends over the rest of the plate and gradually increases in intensity. At 30 volts the blackening is about the same as the high velocity band. There is consequently an appreciable number of electrons which lose practically no velocity when reflected. They have probably collided with a nucleus. A primary electron which collides with another electron in an atom loses a velocity equal to or greater than the equivalent of the ionization potential. On this view we might expect the relative intensity of the high velocity band to decrease with increasing atomic number. It does so for aluminum, brass, and platinum.

10. The mean free path of slow electrons in nitrogen, methane, helium, and argon. ROBERT B. BRODE, California Institute of Technology.—The mean free paths (inverse of effective molecular cross-sections) were observed for electrons of from 2 to 360 volts velocity. The method used was similar to that used by C. Ramsaur (Ann. der Phys.

72, 345, 1923). The very long mean free paths for low velocity electrons in argon observed by Ramsaur, with a minimum at 13.2 volts, were confirmed. A similar phenomenon was observed in methane, with a minimum free path at 6.8 volts and longer mean free paths for electrons of lower velocity. The mean free path of electrons in helium increased steadily with increasing velocity. The mean free path of electrons in nitrogen increased up to 9 volts (resonance potential 8.6 volts), then decreased up to 17 volts (ionization potential 16.2 volts) and thereafter increased steadily.

11. Simultaneous diurnal variation of the electric potential of the earth and the air. FERNANDO SANFORD, Stanford University.—If an elongated insulated conductor be placed vertical anywhere outside of a conducting enclosure, its upper end will be found to have a negative charge and its lower end a positive charge. Such a condition can be maintained only by induction from a positive charge in the atmosphere or a negative charge on the earth. The magnitude of the induced charges upon the insulated conductor is greater at night than in the day and greater in winter than in summer. The inducing charge must accordingly be greater upon that part of the earth which is farthest from the sun. The writer has shown that the night side of the earth is constantly electro-negative to the day side, hence *at any given place* the electric potential of the earth undergoes a diurnal variation. The figures presented with this paper show that the diurnal variation of potential gradient for three months is such as would be expected if it is due to the induction of a negative charge upon the earth.

12. Electrometer variations due to leakage currents. JOSEPH G. BROWN, Stanford University.—If one pair of quadrants of an electrometer is connected to an insulated conductor, and a difference of potential is maintained between the needle and the other pair of quadrants by a battery (all enclosed in a tight metal box), the insulated conductor will in general take a certain charge by leakage through the air or across the insulators. The magnitude of this charge will depend chiefly upon the potentials of near-by parts of the apparatus and the air and insulator resistances, which will determine the rates of leakage. A steady charge can be maintained only when the rates of leakage on and off the conductor remain equal and constant. It has been found that any set-up of the apparatus which depends upon a charge being maintained in this way is subject to needle variations. A set-up in which no charge is maintained in this way is not subject to needle variations. It is inferred that variations in leakage rate, caused by ionization and moisture in the air, will account for these needle variations. Experiments are being carried on which indicate that this inference is correct, but sufficient data have not been obtained to warrant a definite conclusion.

13. A high-voltage direct-current generator. S. S. MACKEOWN, California Institute of Technology.—This generator consists of a Tesla coil excited by a vacuum tube oscillator, the voltage across the Tesla coil being rectified by a General Electric kenetron. The frequency is about 2×10^5 cycles per second. In series with the kenetron is a condenser which is charged to the maximum voltage of the Tesla coil. The high voltage direct current is taken from the terminals of this condenser. The voltage is limited by that which the kenetron will support (100,000 volts). Using a condenser of .006 mf, the ripple will be, theoretically, less than 0.1 per cent with a voltage of 100,000, for any current less than 0.1 ampere, provided the variations in the plate voltage to the vacuum tube do not exceed this limit.

14. A possible manifestation of directional hysteresis in iron. EDWARD CONDON, University of California.—If a soft steel rod be placed within a magnetizing solenoid equipped with a secondary coil and the permeability be measured in the ballistic way, widely differing values are obtained when the rod is traversed by 60-cycle alternating currents of different strengths. Similar results are obtained when a tool steel tube is

substituted for the rod and the current is carried by a wire through, and insulated from, the tube. When the primary field is 0.57 gauss, a current of 60 amperes in the rod magnifies the apparent permeability about 8 times. Currents of 30 amperes cause even greater magnification. A 100-ampere current increases the permeability in the ratio 3 to 1. Increase of temperature increases the zero-current permeability while decreasing the anomalous values; it is found that at 350° C the permeability for 100 amperes is equal to that for zero current. At higher temperatures the former is the lesser. With the tube, maximum apparent permeability occurs with 100 amperes in the central conductor. It is suggested that these effects are a manifestation of directional hysteresis in the magnetic induction caused by the variation in direction of the applied magnetic field.

15. Dispersion and energy distribution in transverse elastic waves. PERRY BYERLY, JR., University of California.—A study of the earthquakes of January 17, 1922, in Venezuela and in Ecuador, and of the earthquake of January 31, 1922, in California, was undertaken to investigate the question of dispersion of transverse elastic waves of the second preliminary group, and the distribution of energy in that group. The first two earthquakes each had a very marked maximum in the *S* group. The second had two marked maxima. It was found that the periods of these maxima had no functional relation to the distance from the epicenter. In each of the first two earthquakes the velocity of the maximum was greater than that of the group. In the California earthquake the velocity of each maximum was less than that of the group. It is concluded that in the second preliminary group, velocity is not a function of the period, i.e., there is no dispersion; the position of the maximum in the group is a function of the distance from the epicenter, but not the same function for all earthquakes.

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